Some solution concepts for strategic games

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Fall 2026

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  A strategy profile s is an ϵ-Nash equilibrium (in the additive sense) if, for all agents i and for all strategies $s_1' \in \Delta(A_i)$

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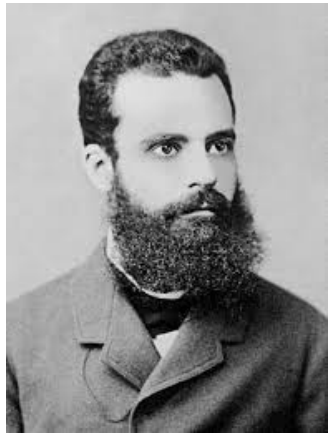
$$U_i(s) \geq (1 - \epsilon) U_i(s_{-i}, s_i')$$

Pareto efficiency

- Pareto efficiency, or Pareto optimality, is a strategy profile in which it is impossible to make any one individual better off without making at least one individual worse off.



- The term is named after Vilfredo Pareto (1848–1923), an Italian engineer and economist who used the concept in his studies of economic efficiency and income distribution.

PE: Definition

- The strategy profile s is Pareto dominated by the strategy profile s' iff $u_i(s') \geq u_i(s)$, for $i \in N$, and there is $j \in N$ with $u_j(s') > u_j(s)$.
- The strategy profile s is Pareto efficient iff it is not Pareto dominated by any other strategy profile.

PE: Example

	Quiet	Fink
Quiet	2,2	0,3
Fink	3,0	1,1

	Bach	Stravinsky
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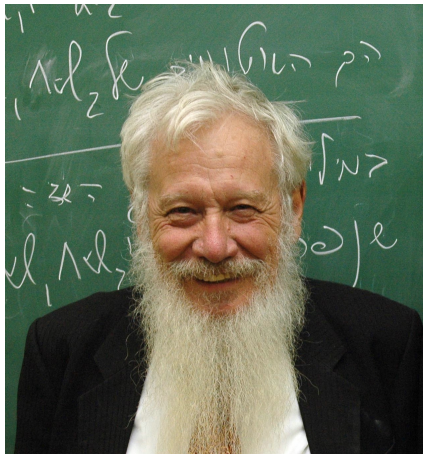
- Prisoners dilemma (Fink,Fink) is not Pareto efficient.
- Bach-Stravinsky the PNE are Pareto efficient.

Correlated equilibrum

- A correlated equilibrium is a solution concept that is more general than the Nash equilibrium.
- It was first discussed by Robert Aumann (1974).



Correlated equilibrum

- A correlated equilibrium is a general distribution over strategy profiles.
- If a strategy profile is drawn from this distribution, and each player is told separately his/her own component, then no player has an incentive to choose a different strategy, because, assuming that all other players also obey, the suggested strategy is the best in expectation.

CE: Definition

- Let S_i be the set of strategies for player i and let S be the set of strategy profiles of Γ.
- A distribution on S is a vector of nonnegative real numbers, one for each strategy profile in S, adding up to 1.
- A correlated equilibrium is a distribution x on S such that for all players i and all strategies $s_i, s_i' \in S_i$ the following is true: Conditioned on the i-th component of a strategy profile drawn from x being s_i, the expected utility for i of playing s_i is no smaller than that of playing s_i':

$$\sum_{s \in S} [u(s_{-i}, s_i) - u(s_{-i}, s_i')] x_i(s).$$

CE: Existence

- A distribution x on S is a product if for each player i there is a distribution x^i on S_i such that for all $s = (s_1, \ldots, s_n) \in S$, $x(s) = \prod_{i \in N} x^i(s_i)$.
- A Nash equilibrium is a correlated equilibrium that happens to be a product distribution.

CE: Example

The chicken game has 2 players (think of them as very competitive drivers speeding from different streets to an intersection), each with two strategies:

	Stop	Go
Stop	4,4	1,5
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x_2	0	0	1	0
x_3	1/4	1/4	1/4	1/4
x_4	0	1/2	1/2	0
x_5	1/3	1/3	1/3	0

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x_1 and x_2 correspond to the two PNE of the game.
x_3 is the only other NE.
x_4 implements a fair traffic light.
x_5 is the correlated equilibrium that maximizes expected utility.