1. In a conflict analysis, let $C \lor l$ be the first clause (obtained by resolving propagated literals with their reason clauses) that contains exactly one literal $l$ of the current decision level ($C \lor l$ is usually called the first unique implication point, denoted 1UIP). Prove that, if further propagated literals of the current decision level are resolved and another clause $C' \lor l'$ with exactly one literal $l'$ of the current decision level is obtained, then $\max_{\ell \in C_{\text{dec level}}} \text{dec level}(\ell) \leq \max_{\ell \in C'_{\text{dec level}}} \text{dec level}(\ell)$.

2. The problem of AllSAT consists in obtaining all models of a given propositional formula. Design an algorithm for AllSAT on top of a CDCL SAT solver.

3. A binary clause is a clause with at most two literals. Given $S$ a set of binary clauses defined over the propositional symbols $p_1, ..., p_n$, the graph associated to $S$, denoted $G_S$, is the directed graph $G_S = (V, E)$, where $V = \{p_1, \ldots, p_n, \neg p_1, \ldots, \neg p_n\}$ and $E = \{(l, l') \mid \neg l \lor l' \in S\}$ (unit clauses, i.e. of the form $p$, are considered as $p \lor p$).
   
   (a) Show that if there is a path from $l$ to $l'$ in $G_S$, then $\neg l \lor l' \in \text{Res}(S)$. Conversely, show that if $l \lor l' \in \text{Res}(S)$, then there is a path from $\neg l$ to $l'$ in $G_S$.
   
   (b) Show that $S$ is unsatisfiable if and only if there exists a propositional symbol $p$ such that there is a path in $G_S$ from $p$ to $\neg p$, and another one from $\neg p$ to $p$.
   
   (c) Design an algorithm for deciding the satisfiability of a set of binary clauses. Which is its complexity, in terms of the number of clauses and the number of propositional symbols?

4. Given an unsatisfiable set of clauses, explain how to instrument a SAT solver so as to obtain a refutation by resolution.