Tutorial on CPLEX
Linear Programming

Combinatorial Problem Solving (CPS)

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Among other things, CPLEX allows one to deal with:

- **Real linear progs** (all vars are in $\mathbb{R}$)

  $\begin{align*}
  \text{min } & \ c^T x \\
  & A_1 x \leq b_1 \\
  & A_2 x = b_2 \\
  & A_3 x \geq b_3 \\
  & x \in \mathbb{R}^n
  \end{align*}$

- **Mixed integer linear progs** (some vars are in $\mathbb{Z}$)

  $\begin{align*}
  \text{min } & \ c^T x \\
  & A_1 x \leq b_1 \\
  & A_2 x = b_2 \\
  & A_3 x \geq b_3 \\
  & \forall i \in I : x_i \in \mathbb{Z} \\
  & \forall i \notin I : x_i \in \mathbb{R}
  \end{align*}$
**CPLEX Toolkit**

- CPLEX allows one to work in several ways. CPLEX is...
  - An IDE that uses the OPL modeling language
  - An interactive optimizer that reads MPS/LP input
  - A callable library in several languages
    - Java
    - C
    - C++ *(Concert Technology)*
    - ...

Two kinds of objects:

- **Modeling objects** for defining the optimization problem (constraints, objective function, etc.)

  They are grouped into an `IloModel` object representing the complete optimization problem (recall: here, `model = problem`).

- **Solving objects** for solving problems represented by modeling objects.

  An `IloCplex` object reads a model, extracts its data, solves the problem and answers queries on solution.
Creating the Environment: IloEnv

- The class IloEnv constructs a CPLEX environment.
- The environment is the first object created in an application.
- To create an environment named env, you do this:

```c
IloEnv env;
```

- The environment object needs to be available to the constructor of all other Concert Technology classes
- IloEnv is a handle class: variable env is a pointer to an implementation object, which is created at the same time
- Before terminating destroy the implementation object with

```c
env.end();
```

for just ONE of its IloEnv handles
Creating a Model: IloModel

- After creating the environment, a Concert application is ready to create one or more optimization models.
- Objects of class IloModel define a complete model that can be later passed to an IloCplex object.
- To construct a modeling object named model, within an existing environment named env, call:

  ```cpp
  IloModel model(env);
  ```

- The environment of a given optimization model can be recovered by calling:

  ```cpp
  IloEnv env = model.getEnv();
  ```
Creating a Model: IloModel

- After an IloModel object has been constructed, it can be populated with objects of classes:
  - IloNumVar, representing modeling variables;
  - IloRange, which define constraints of the form $l \leq E \leq u$, where $E$ is a linear expression;
  - IloObjective, representing an objective function.

- Any object obj can be added to the model by calling:

```java
model.add(obj);
```

- No need to explicitly add the variable objects to a model, as they are implicitly added when they are used in range constraints (instances of IloRange) or in the objective.

- At most one objective can be used in a model.
Creating a Model: IloModel

- Modeling variables are constructed as objects of class IloNumVar, e.g.:

  ```java
  IloNumVar x(env, 0, 40, ILOFLOAT);
  ```

  This definition creates the modeling variable `x` with lower bound 0, upper bound 40 and type ILOFLOAT

- Variable types:
  - ILOFLOAT: continuous variable
  - ILOINT: integer variable
  - ILOBOOL: Boolean variable
Creating a Model: IloModel

- After all the modeling variables have been constructed, they can be used to build expressions, which are used to define objects of classes IloObjective, IloRange.

- To create obj of type IloObjective representing an objective function (and direction of optimization):

\[
\text{IloObjective obj = IloMinimize(env, x+2*y);} \\
\]

- Creating constraints and adding them to the model:

\[
\text{model.add(-x + 2*y + z} <= 20); \\
\]

\[
-x + 2*y + z <= 20 \text{ creates implicitly an object of class IloRange that is immediately added to the model} \\
\]

- One may have arrays of these objects: IloNumVarArray, IloRangeArray
Creating a Model: IloModel

- Actually in

```cpp
model.add(-x + 2*y + z <= 20);
```

an object of class IloExpr is also implicitly created.

- Objects of class IloExpr can be created explicitly too.
  E.g., when expressions cannot be spelled out in source code but have to be built up dynamically. Operators like `+=` provide an efficient way to do this.

- IloExpr objects are handles.
  So the method `end()` must be called when the object is no longer needed.

  The only exception to this rule are implicit expressions, where user does not create an IloExpr object explicitly (see the example).
Solving the Model: IloCplex

- The class IloCplex solves a model.
- After the optimization problem has been stored in an IloModel object (say, model), it is time to create an IloCplex object (say, cplex) for solving the problem:

  ```
  IloCplex cplex(model);
  ```

- To solve the model, call:

  ```
  cplex.solve();
  ```

- This method returns an IloBool value, where:
  - IloTrue indicates that CPLEX successfully found a feasible (yet not necessarily optimal) solution
  - IloFalse indicates that no solution was found
Solving the Model: IloCplex

- More precise information about the outcome of the last call to the method `solve` can be obtained by calling:

```java
cplex.getStatus();
```

- Returned value tells what CPLEX found out: whether
  - it found the optimal solution or only a feasible one; or
  - it proved the model to be unbounded or infeasible; or
  - nothing at all has been proved at this point.

- More info is available with method `getCplexStatus`. 
Querying Results

- Query methods access information about the solution.
- Numbers in solution, etc. are of type IloNum
- To query the solution value for a variable:

  ```
  IloNum v = cplex.getValue(x);
  ```

- To query the solution value for an array of variables:

  ```
  IloNumVarArray x(env);
  ...
  IloNumArray v(env);
  cplex.getValues(v, x);
  ```
■ To get the values of the slacks of an array of constraints:

```cpp
IloRangeArray c(env);
...
IloNumArray v(env);
cplex.getSlacks(v, c);
```

■ To get the values of the duals of an array of constraints:

```cpp
IloRangeArray c(env);
...
IloNumArray v(env);
cplex.getDuals(v, c);
```
Querying Results

- To get values of reduced costs of an array of variables:

```java
IloNumVarArray x(env);
...
IloNumArray v(env);
cplex.getReducedCosts(v, x);
```

- To avoid logging messages by CPLEX on screen:

```java
cplex.setOut(env.getNullStream());
```
Querying Results

- Output operator `<<` is defined for type `IloAlgorithm::Status` returned by `getStatus`, as well as for `IloNum`, `IloNumVar`, ...

  `<<` is also defined for any array of elements if the output operator is defined for the elements.

- Default names are of the form `IloNumVar(\(n\))[\(\ell..u\)]` for variables, and similarly for constraints, e.g.,

  \[
  \text{IloNumVar}(1)[0..9] + \text{IloNumVar}(3)[0..\text{inf}] \leq 20
  \]

- One can set names to variables and constraints:

  ```
  x.setName("x");
  c.setName("c");
  ```
Writing/Reading Models

- CPLEX supports reading models from files and writing models to files in several languages (e.g., LP format, MPS format)
- To write the model to a file (say, model.lp):

```cplex.exportModel("model.lp");```

- IloCplex decides which file format to write based on the extension of the file name (e.g., .lp is for LP format)
- This may be useful, for example, for debugging
Languages for Linear Programs

- **MPS**
  - Very old format (≈ age of punched cards!) by IBM
  - Has become industry standard over the years
  - Column-oriented
  - Not really human-readable nor comfortable for writing
  - All LP solvers support this language

- **LP**
  - CPLEX specific file format
  - Row-oriented
  - Very readable, close to mathematical formulation
  - Supported by CPLEX, GUROBI, GLPK, LP_SOLVE, ..
    (which can translate from one format to the other!)
Example: Product Mix Problem

- A company can produce 6 different products $P_1, \ldots, P_6$
- Production requires labour, energy and machines, which are all limited
- The company wants to maximize revenue
- The next table describes the requirements of producing one unit of each product, the corresponding revenue and the availability of resources:

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Machine</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1050</td>
</tr>
<tr>
<td>Labour</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1050</td>
</tr>
<tr>
<td>Energy</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>1080</td>
</tr>
</tbody>
</table>
Example: Product Mix Problem

MODEL:

\[ x_i = \text{quantity of product } P_i \text{ to be produced.} \]

\[
\begin{align*}
\text{max Revenue:} & \quad 5x_1 &+ 6x_2 &+ 7x_3 &+ 5x_4 &+ 6x_5 &+ 7x_6 \\
\text{Machine:} & \quad 2x_1 &+ 3x_2 &+ 2x_3 &+ x_4 &+ x_5 &+ 3x_6 &\leq 1050 \\
\text{Labour:} & \quad 2x_1 &+ x_2 &+ 3x_3 &+ x_4 &+ 3x_5 &+ 2x_6 &\leq 1050 \\
\text{Energy:} & \quad 1x_1 &+ 2x_2 &+ x_3 &+ 4x_4 &+ x_5 &+ 2x_6 &\leq 1080 \\
& \quad x_1, & x_2, & x_3, & x_4, & x_5, & x_6 &\geq 0
\end{align*}
\]
\ Product-mix problem (LP format)

\[ \text{max} \]
\[ \text{revenue: } 5 \ x_1 \ + \ 6 \ x_2 \ + \ 7 \ x_3 \ + \ 5 \ x_4 \ + \ 6 \ x_5 \ + \ 7 \ x_6 \]

\text{subject to}

\[ \text{machine: } 2 \ x_1 \ + \ 3 \ x_2 \ + \ 2 \ x_3 \ + \ x_4 \ + \ x_5 \ + \ 3 \ x_6 \ \leq \ 1050 \]
\[ \text{labour: } 2 \ x_1 \ + \ x_2 \ + \ 3 \ x_3 \ + \ x_4 \ + \ 3 \ x_5 \ + \ 2 \ x_6 \ \leq \ 1050 \]
\[ \text{energy: } 1 \ x_1 \ + \ 2 \ x_2 \ + \ x_3 \ + \ 4 \ x_4 \ + \ x_5 \ + \ 2 \ x_6 \ \leq \ 1080 \]

\text{end}
MPS Format

* Product-mix problem (Fixed MPS format)
*
* Column indices
*000000001111111111222222222233333333334444444445555555555666666666666666
*23456789012345678901234567890123456789012345678901234567890123456789
*
* mrevenue stands for -revenue
*
NAME PRODMIX
ROWS
N mrevenue
L machine
L labour
L energy
COLUMNS
x_1 mrevenue -5 machine 2
x_1 labour 2 energy 1
x_2 mrevenue -6 machine 3
x_2 labour 1 energy 2
x_3 mrevenue -7 machine 2
x_3 labour 3 energy 1
x_4 mrevenue -5 machine 1
x_4 labour 1 energy 4
x_5 mrevenue -6 machine 1
x_5 labour 3 energy 1
x_6 mrevenue -7 machine 3
x_6 labour 2 energy 2
RHS
RHS1 machine 1050 labour 1050
RHS1 energy 1080
ENDATA
LP Format

- Intended for representing LP’s of the form

\[
\begin{align*}
\min / \max & \quad c^T x \\
{a_i^T x \bowtie_i b_i} & \quad (1 \leq i \leq m, \bowtie_i \in \{\leq, =, \geq\}) \\
\ell \leq x \leq u & \quad (-\infty \leq \ell_k, u_k \leq +\infty)
\end{align*}
\]

- Comments: anything from a backslash \ to end of line
- In general blank spaces are ignored (except for separating keywords)
- Names are sequences of alphanumeric chars and symbols ( , ) _ etc. Careful with e, E: troubles with exponential notation!
LP Format

1. Objective function section

(a) One of the keywords: min, max
(b) Label with semi-colon: e.g. cost: (optional)
(c) Expression: e.g. $-2 \, x_1 + 2 \, x_2$

2. Constraints section

(a) Keyword subject to (or equivalently: s.t., st, such that)
(b) List of constraints, each in a different line
   i. Label with semi-colon: e.g. limit: (optional)
   ii. Expression: e.g. $3 \, x_1 + 2 \, x_2 \leq 4$
      Senses: $\leq$, $<=$ for $\leq$; $\geq$, $=>$ for $\geq$; $=$ for $=$
LP Format

3. Bounds section (optional)

(a) Keyword **Bounds**

(b) List of bounds, each in a different line

\[
\begin{align*}
1 & \leq x \leq u: & \text{sets lower and upper bounds} \\
1 & \leq x: & \text{sets lower bound} \\
x & \geq 1: & \text{sets lower bound} \\
x & \leq u: & \text{sets upper bound} \\
x & = f: & \text{sets a fixed value} \\
x & \text{free:} & \text{specifies a free variable}
\end{align*}
\]

(c) Infinite bounds $-\infty$, $+\infty$ are represented $-\text{inf}$, $+\text{inf}$

(d) Default bounds: lower bound 0, upper bound $+\infty$

4. Generals section: Keyword **Generals** + list of integer variables (optional)

5. Binary section: Keyword **Binary** + list of binary variables (optional)

6. End section: File should end with keyword **End**
IloCplex supports reading files with `importModel`

A call to `importModel` causes CPLEX to read a problem from a file and add all data in it as new objects.

```cpp
void IloCplex::importModel (    IloModel& m,   const char* filename,   IloObjective& obj,   IloNumVarArray vars,   IloRangeArray rngs ) const;
```
Example 1

Let us see a program for solving:

\[
\begin{align*}
\text{max} & \quad x_0 + 2x_1 + 3x_2 \\
-x_0 + x_1 + x_2 & \leq 20 \\
x_0 - 3x_1 + x_2 & \leq 30 \\
0 & \leq x_0 \leq 40 \\
0 & \leq x_1 \leq \infty \\
0 & \leq x_2 \leq \infty \\
x_i & \in \mathbb{R}
\end{align*}
\]
# include <ilcplex/ilcplex.h>
ILOSTLBEGIN
int main () {
    IloEnv env;
    IloModel model(env);
    IloNumVarArray x(env);
    x.add(IloNumVar(env, 0, 40));
    x.add(IloNumVar(env));//default: between 0 and +\infty
    x.add(IloNumVar(env));
    model.add(-x[0] + x[1] + x[2] <= 20);
    model.add(x[0] - 3 * x[1] + x[2] <= 30);
    model.add(IloMaximize(env, x[0]+2*x[1]+3*x[2]));
    IloCplex cplex(model);
    cplex.solve();
    cout << "Max=" << cplex.getObjValue() << endl;
    env.end();
}
Example 2

Let us see a program for solving:

\[
\begin{align*}
\text{max} & \quad x_0 + 2x_1 + 3x_2 + x_3 \\
- & x_0 + x_1 + x_2 + 10x_3 \leq 20 \\
& x_0 - 3x_1 + x_2 \leq 30 \\
& x_1 - 3.5x_3 = 0 \\
& 0 \leq x_0 \leq 40 \\
& 0 \leq x_1 \leq \infty \\
& 0 \leq x_2 \leq \infty \\
& 2 \leq x_3 \leq 3 \\
x_0, x_1, x_2 & \in \mathbb{R} \\
x_3 & \in \mathbb{Z}
\end{align*}
\]
#include <ilcplex/ilocplex.h>
ILOSTLBEGIN
int main () {
    IloEnv env;
    IloModel model(env);
    IloNumVarArray x(env);
    x.add(IloNumVar(env, 0, 40));
    x.add(IloNumVar(env));
    x.add(IloNumVar(env));
    x.add(IloNumVar(env, 2, 3, ILOINT));
    model.add( x[0] - 3 * x[1] + x[2] <= 30);
    model.add( x[1] - 3.5* x[3] == 0);
    model.add(IloMaximize(env, x[0]+2*x[1]+3*x[2]+x[3]));
    IloCplex cplex(model); cplex.solve();
    cout << "Max=" << cplex.getObjVal() << endl;
    env.end();
}

More information

- You can find complete documentation in the WWW at:
  
  http://www-01.ibm.com/support/knowledgecenter/SS9UKU

- You can find collection of examples in lab’s machines at:
  /opt/ibm/ILOG/CPLEX_Studio124/cplex/examples/src/cpp
  /opt/ibm/ILOG/CPLEX_Studio124/cplex/examples/data

- You can find a template for Makefile and the examples shown here at:
  www.cs.upc.edu/~erodri/webpage/cps/lab/lp/tutorial-plex-code/tutorial-plex-code.tgz