# Declarative Characterization of a General Architecture for Constructive Geometric Constraint Solvers

Robert Joan-Arinyo

Antoni Soto-Riera

Sebastià Vila-Marta

Josep Vilaplana-Pastó

Departament de Llenguatges i Sistemes Informàtics
Universitat Politècnica de Catalunya

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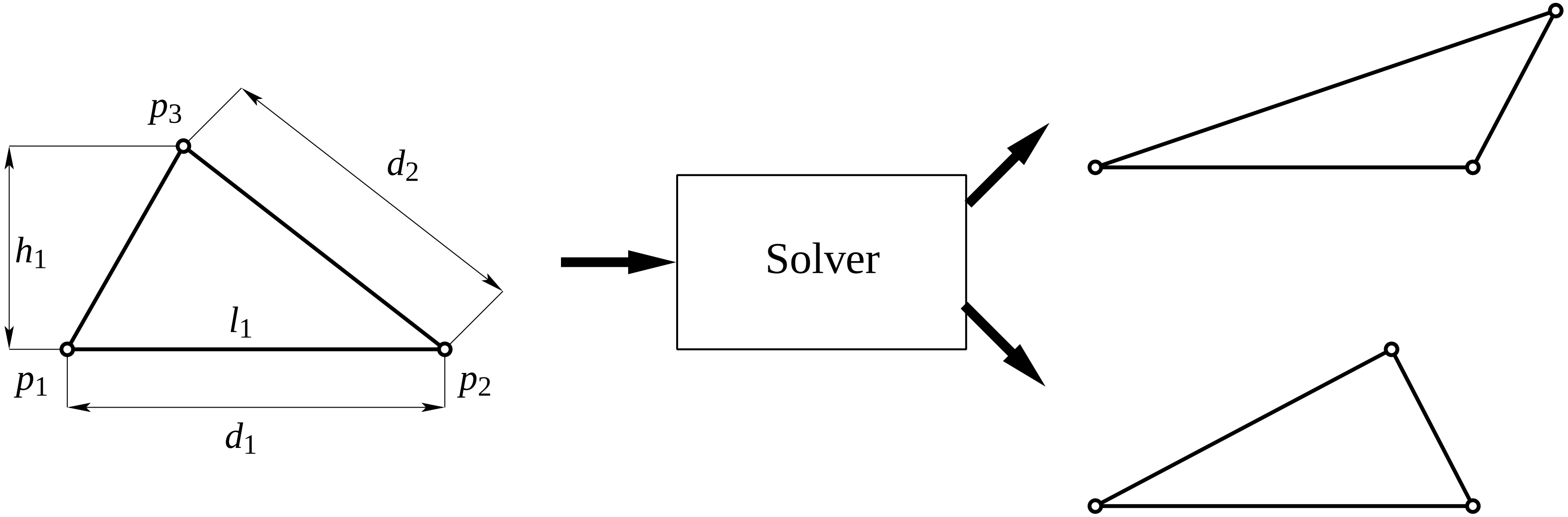
# Geometric constraint solvers

- Geometric constraint problems = geometric elements + constraints between them.

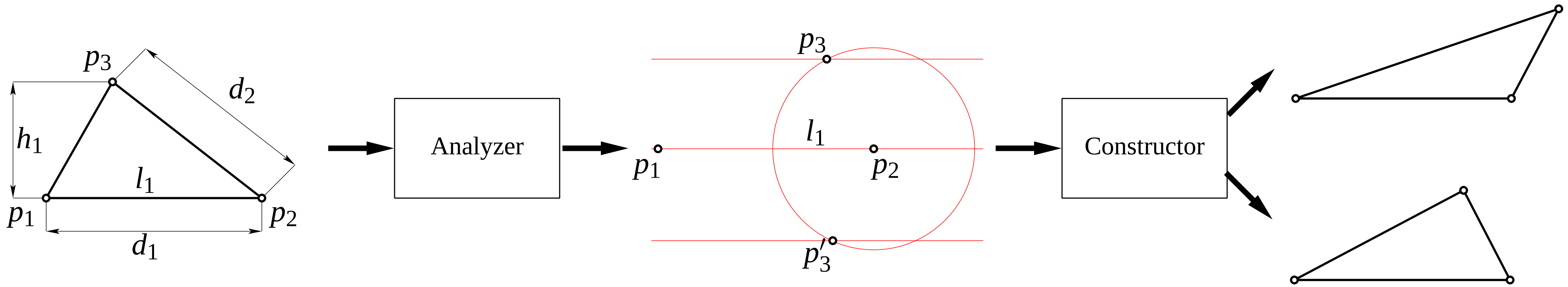
- Geometric constraint solvers analyze geometric problems and construct realizations.



# Constructive geometric constraint solvers

- Constructive geometric constraint solvers describe the solution as a sequence of ruler-and-compass geometric operations.

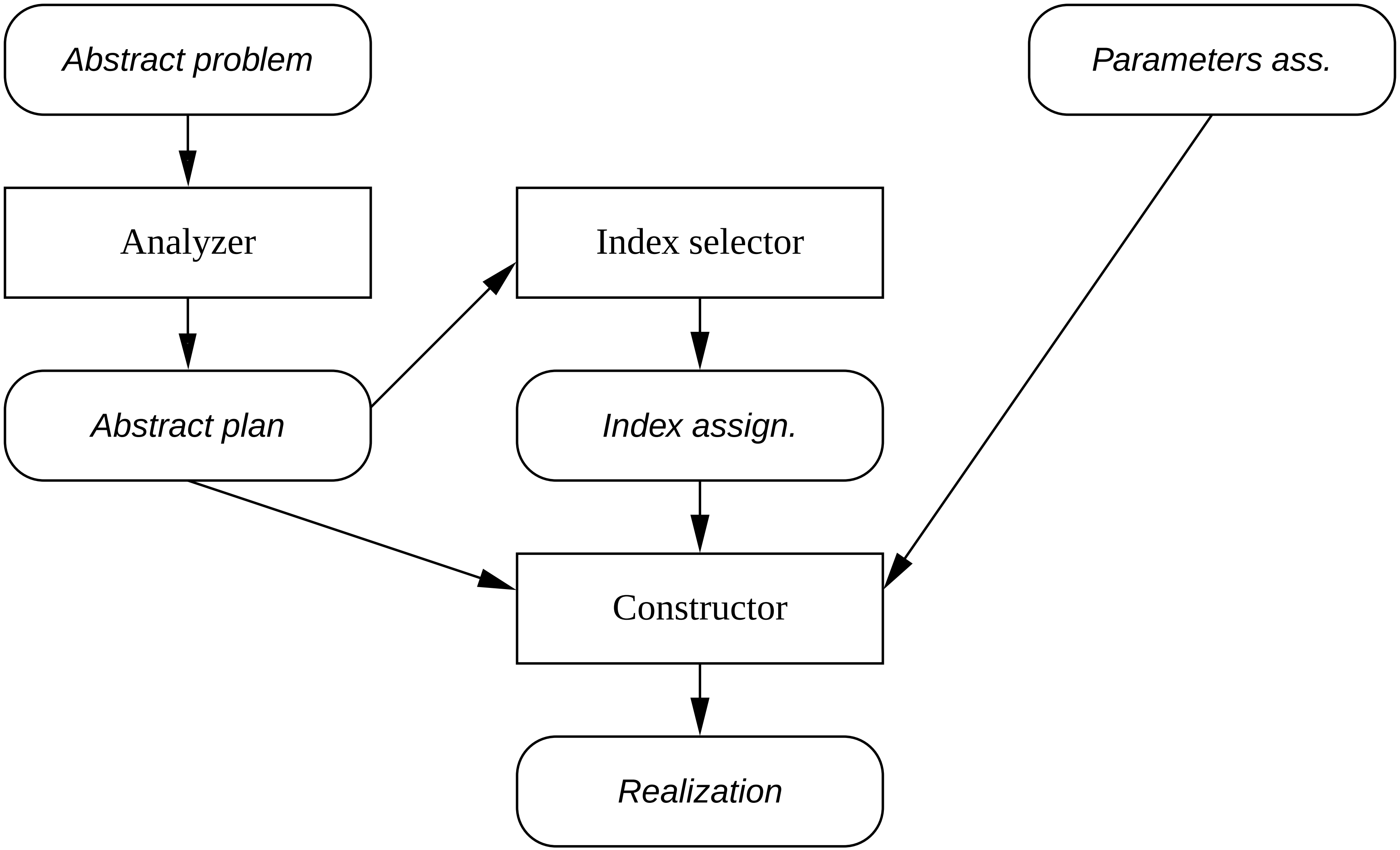
- Constructive geometric constraint solvers share a common architecture.



## Objective: characterize a general architecture for constructive geometric constraint solvers

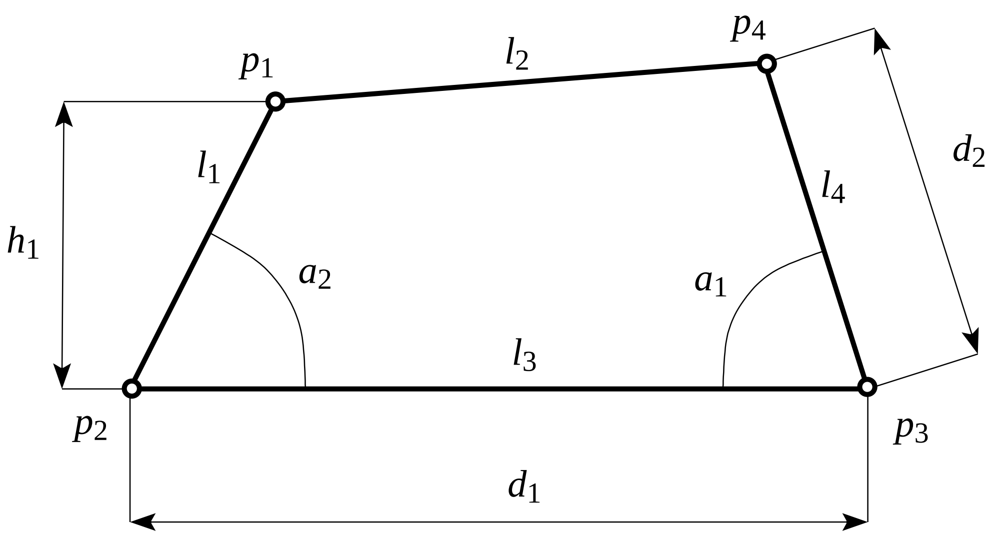
- Identify the relevant functional units.
- Identify the data entities that the functional units manipulate.
- Characterize the functional units independently of its implementation.
- State the semantics of the data entities.

# Architecture overview: a data flow diagram relating data entities and functional units



## Abstract problems

- An abstract problem $A = \langle G, C, P \rangle$ describes the geometric elements $G$, the constraints $C$ and the parameters $P$ of the problem.




$$
\begin{aligned}
G &= \{p_1, p_2, p_3, p_4, l_1, l_2, l_3, l_4\} \\
P &= \{d_1, d_2, a_1, a_2, h_1\} \\
C &= \{onPL(p_1, l_1), \\
&\quad onPL(p_1, l_2), \\
&\quad onPL(p_2, l_1), \\
&\quad onPL(p_2, l_3), \\
&\quad onPL(p_3, l_3), \\
&\quad onPL(p_3, l_4), \\
&\quad onPL(p_4, l_2), \\
&\quad onPL(p_4, l_4), \\
&\quad distPP(p_2, p_3, d_1), \\
&\quad distPP(p_3, p_4, d_2), \\
&\quad distPL(p_1, l_3, h_1), \\
&\quad angleLL(l_3, l_1, a_2), \\
&\quad angleLL(l_3, l_4, a_1)\}
\end{aligned}
$$

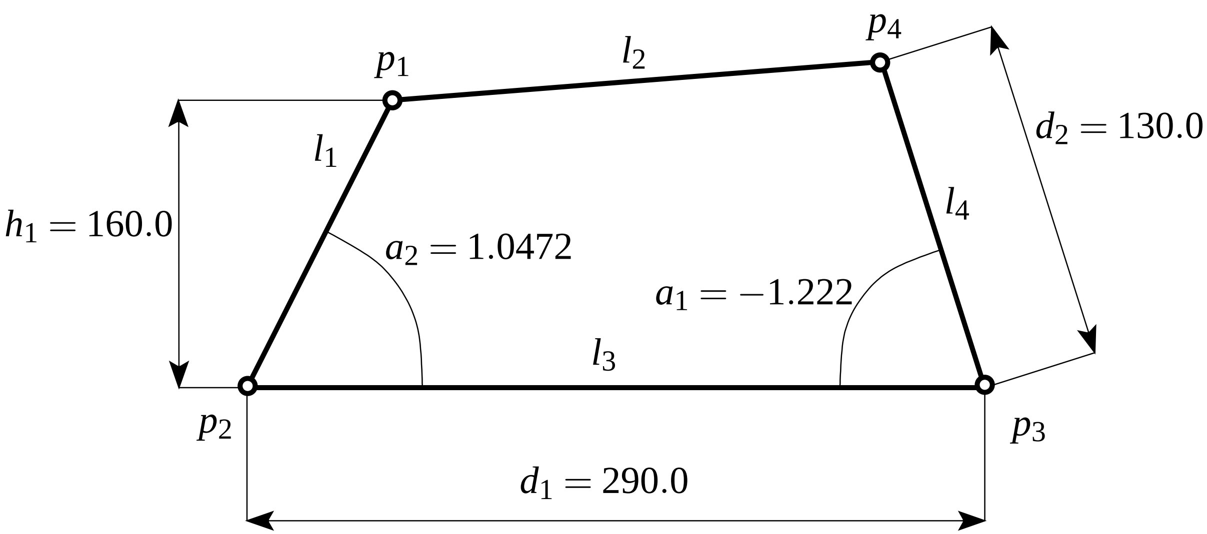
# What does an abstract problem mean?

- Characteristic formula $\Psi$.

$$
\begin{aligned}
\Psi(A) \quad \equiv \quad & (onPL(p_1, l_1) \\
\wedge \quad & onPL(p_1, l_2) \\
\wedge \quad & onPL(p_2, l_1) \\
\wedge \quad & onPL(p_2, l_3) \\
\wedge \quad & onPL(p_3, l_3) \\
\wedge \quad & onPL(p_3, l_4) \\
\wedge \quad & onPL(p_4, l_2) \\
\wedge \quad & onPL(p_4, l_4) \\
\wedge \quad & distPP(p_2, p_3, d_1) \\
\wedge \quad & distPP(p_3, p_4, d_2) \\
\wedge \quad & distPL(p_1, l_3, h_1) \\
\wedge \quad & angleLL(l_3, l_1, a_2) \\
\wedge \quad & angleLL(l_3, l_4, a_1))
\end{aligned}
$$

# Parameters assignments

- A *parameters assignment* $\alpha$ assigns values to parameters symbols.




$$\begin{aligned}
\alpha(a_1) &= -1.222 \\
\alpha(a_2) &= 1.0472 \\
\alpha(h_1) &= 160.0 \\
\alpha(d_1) &= 290.0 \\
\alpha(d_2) &= 130.0
\end{aligned}$$

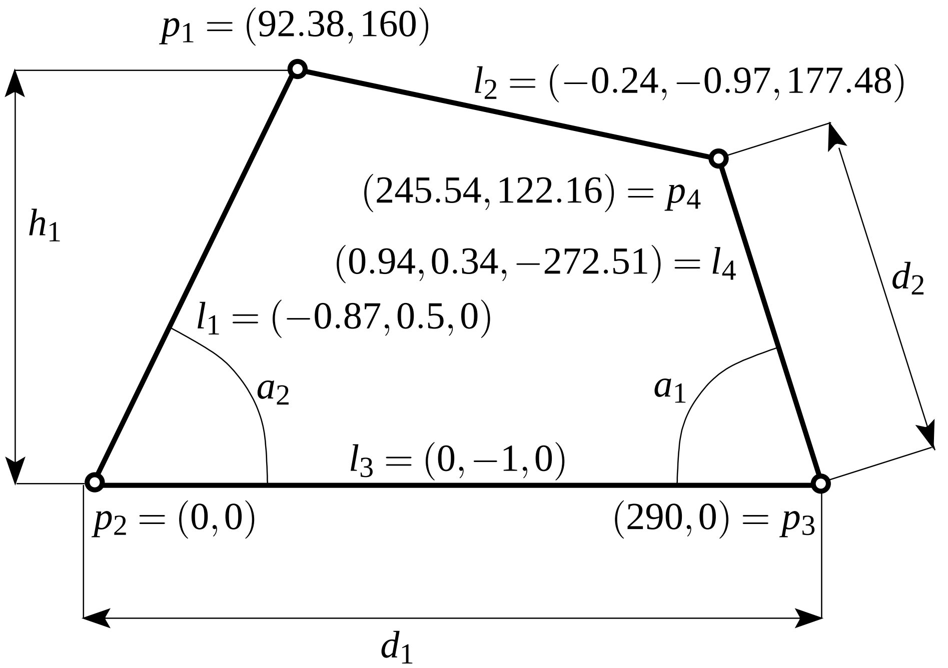
# Instance problems

- $\alpha.A = \langle G, \alpha.C, P \rangle$ is an *instance problem*.

$$
\begin{aligned}
\alpha.C \;=\; \{ & onPL(p_1, l_1), \\
& onPL(p_1, l_2), \\
& onPL(p_2, l_1), \\
& onPL(p_2, l_3), \\
& onPL(p_3, l_3), \\
& onPL(p_3, l_4), \\
& onPL(p_4, l_2), \\
& onPL(p_4, l_4), \\
& distPP(p_2, p_3, 290.0), \\
& distPP(p_3, p_4, 130.0), \\
& distPL(p_1, l_3, 160.0), \\
& angleLL(l_3, l_1, 1.0472), \\
& angleLL(l_3, l_4, -1.222) \}
\end{aligned}
$$

# Geometry assignments

- A *geometry assignment* $\kappa$ assigns coordinates to geometric elements.




$$
\begin{aligned}
\kappa(p_1) &= (92.38, 160) \\
\kappa(p_2) &= (0, 0) \\
\kappa(p_3) &= (290, 0) \\
\kappa(p_4) &= (245.54, 122.16) \\
\kappa(l_1) &= (-0.87, 0.5, 0) \\
\kappa(l_2) &= (-0.24, -0.97, 177.48) \\
\kappa(l_3) &= (0, -1, 0) \\
\kappa(l_4) &= (0.94, 0.34, -272.51)
\end{aligned}
$$

# Which are the solutions of an abstract problem?

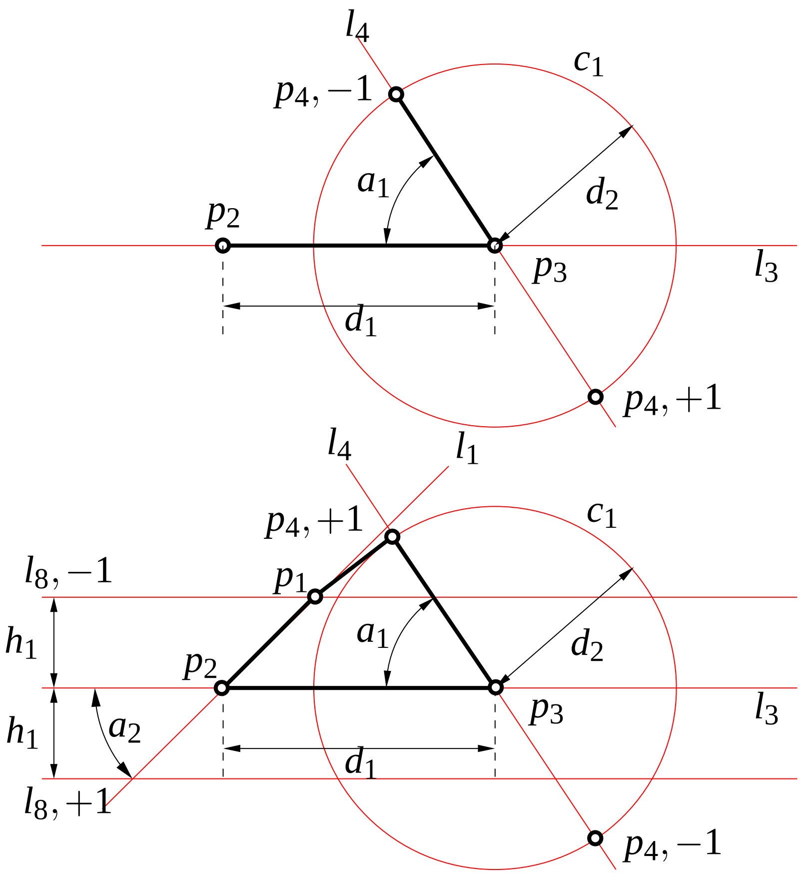
- A *realization* of an instance problem $\alpha.A$ is a geometry assignment $\kappa$ for which the formula $\Psi(\kappa.\alpha.A)$ holds.

- $V(\alpha.A)$ is the set of realizations of the instance problem $\alpha.A$.

$$V(\alpha.A) = \{\kappa \quad | \quad \Psi(\kappa.\alpha.A)\}$$

# Abstract plans

- An abstract plan $S = \langle G, P, L, I \rangle$ is a sequence of geometric operations $L$ that computes the coordinates of the geometric elements in $G$.




$$
\begin{aligned}
L \;=\; \{ & p_2 = pointXY(O_x, O_y) \\
& p_3 = pointXY(d_1, O_y) \\
& c_1 = circleCR(p_3, d_2) \\
& l_3 = linePP(p_2, p_3) \\
& l_4 = lineAP(l_3, a_1, p_3) \\
& p_4 = interLC(l_4, c_1, s_1) \\
& l_1 = lineAP(l_3, a_2, p_2) \\
& l_8 = lineLD(l_3, h_1, s_2) \\
& p_1 = interLL(l_1, l_8) \\
& l_2 = linePP(p_1, p_4) \}
\end{aligned}
$$

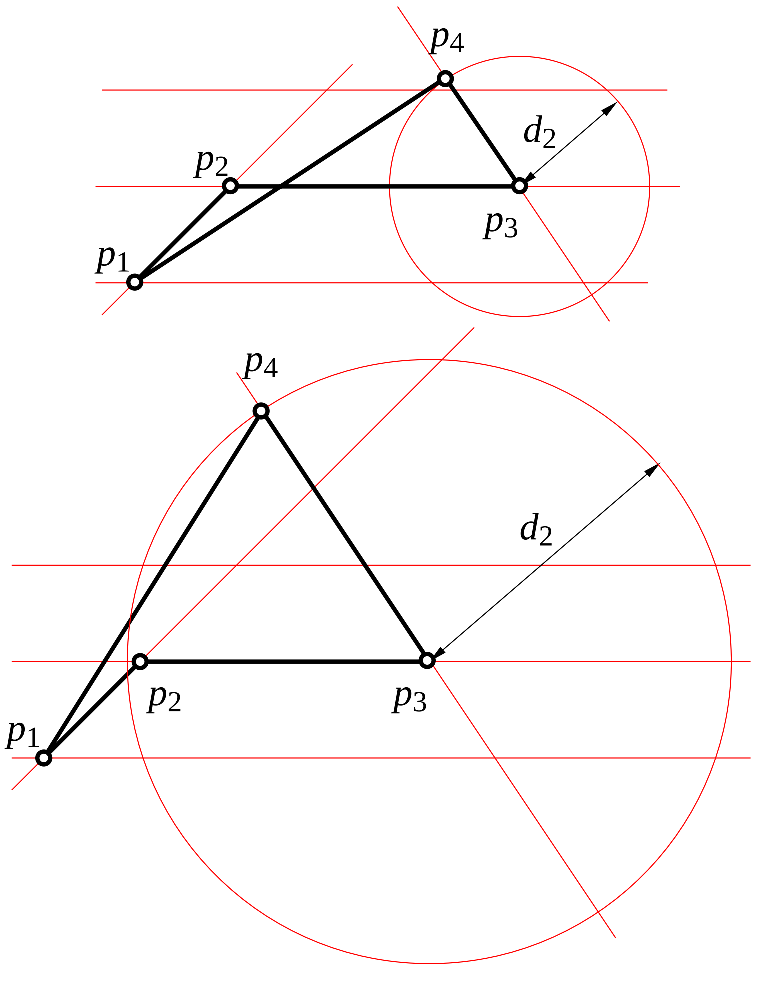
# What does an abstract plan mean?

- Characteristic formula $\Phi$.

$$
\begin{aligned}
\Phi(S) \quad \equiv \quad & (p_2 = pointXY(O_x, O_y) \\
\wedge \quad & \; p_3 = pointXY(d_1, O_y) \\
\wedge \quad & \; c_1 = circleCR(p_3, d_2) \\
\wedge \quad & \; l_3 = linePP(p_2, p_3) \\
\wedge \quad & \; l_4 = lineAP(l_3, a_1, p_3) \\
\wedge \quad & \; p_4 = interLC(l_4, c_1, s_1) \\
\wedge \quad & \; l_1 = lineAP(l_3, a_2, p_2) \\
\wedge \quad & \; l_8 = lineLD(l_3, h_1, s_2) \\
\wedge \quad & \; p_1 = interLL(l_1, l_8) \\
\wedge \quad & \; l_2 = linePP(p_1, p_4))
\end{aligned}
$$

# Index assignments

- An *index assignment* $\iota$ assigns values to the sign parameters in the index.




$$\iota(s_1) = +1$$
$$\iota(s_2) = +1$$

# Instance plans and indexed plans

- $\alpha.S = \langle G, P, \alpha.L, I \rangle$ is an *instance plan*.
- $\iota.S = \langle G, P, \iota.L, I \rangle$ is an *indexed plan*.
- An example of $\iota.\alpha.S$.

$$
\begin{array}{rcl}
\iota.\alpha.L & = & \{ p_2 = pointXY(O_x, O_y) \\
& \wedge & \quad p_3 = pointXY(290.0, O_y) \\
& \wedge & \quad c_1 = circleCR(p_3, 130.0) \\
& \wedge & \quad l_3 = linePP(p_2, p_3) \\
& \wedge & \quad l_4 = lineAP(l_3, -1.222, p_3) \\
& \wedge & \quad p_4 = interLC(l_4, c_1, +1) \\
& \wedge & \quad l_1 = lineAP(l_3, 1.0472, p_2) \\
& \wedge & \quad l_8 = lineLD(l_3, 160.0, +1) \\
& \wedge & \quad p_1 = interLL(l_1, l_8) \\
& \wedge & \quad l_2 = linePP(p_1, p_4) \}
\end{array}
$$

# Which are the solutions of an abstract plan?

- An *indexed anchor* of an instance plan $\alpha.S$ is a geometry assignment $\kappa$ for which there is an index assignment $\iota$ such that the formula $\Phi(\kappa.\iota.\alpha.S)$ holds.

- $V(\alpha.S)$ is the set of indexed anchors of the instance plan $\alpha.S$.

$$V(\alpha.S) = \{\kappa \quad | \quad \exists \iota \, \Phi(\iota.\kappa.\alpha.S)\}$$

# Analyzers

- An *analyzer* computes an abstract plan $S = \langle G, P, L, I \rangle$ from an abstract problem $A = \langle G, C, P \rangle$.


- *Correct* analyzers compute construction plans that generate realizations when carried out.

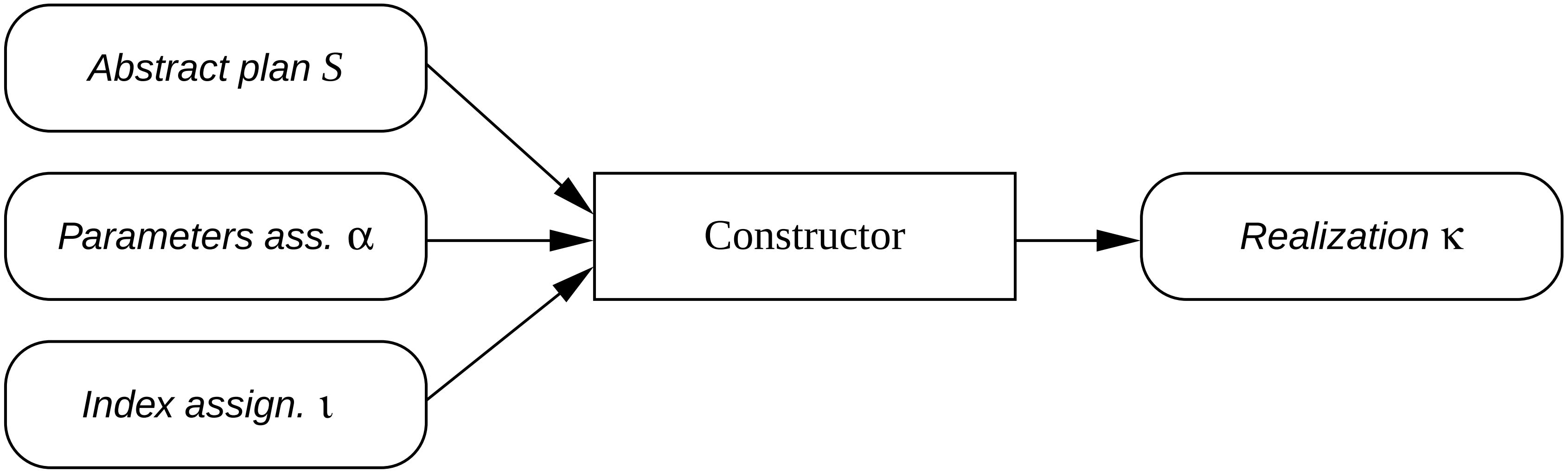
$$V(\alpha.S) \subseteq V(\alpha.A)$$

- *Complete* analyzers compute construction plans that generate exactly the set of realizations when carried out.

$$V(\alpha.S) = V(\alpha.A)$$

# Constructors

- A *constructor* carries out the geometric operations in an abstract plan $S = \langle G, P, L, I \rangle$.

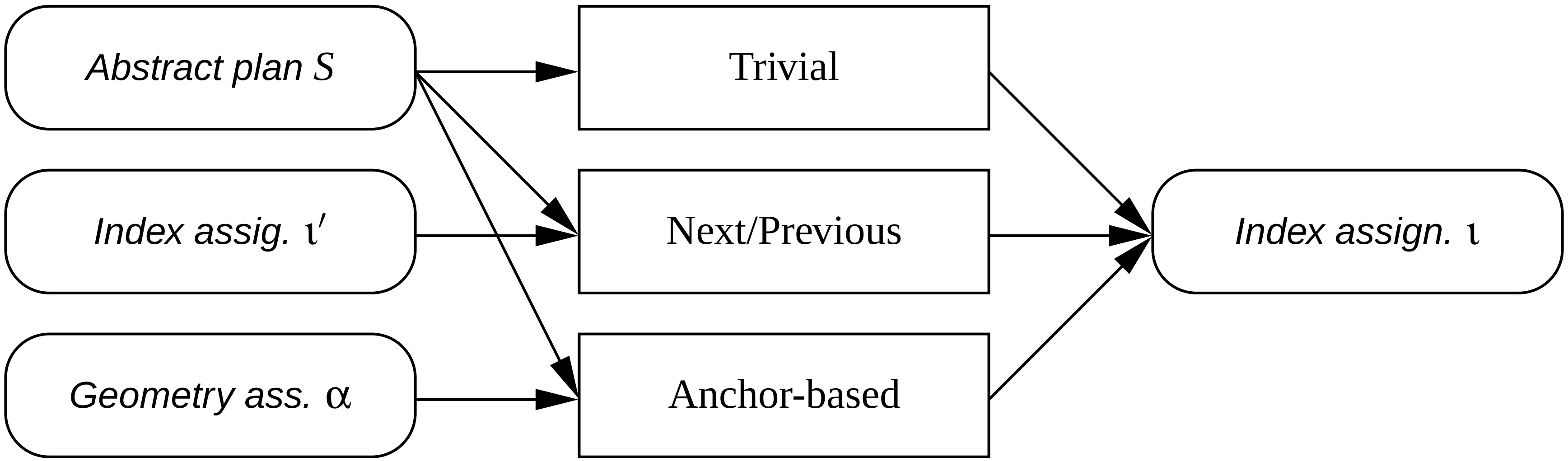



- The geometric assignment $\kappa$ is a realization provided than the analyzer which has computed $S$ is correct.

- $\kappa$ is such that $\Phi(\kappa.\iota.\alpha.S)$ holds.

# Index selectors

- An *index selector* selects a unique solution among a possibly exponential number of solutions described in the construction plan.




- The anchor-based index selector computes an index assignment $\iota$ such that $\Phi(\iota.\kappa.\alpha_\kappa.S)$ holds. $\kappa$ is a geometry assignment and $\alpha_\kappa$ is a parameters assignment obtained from $\kappa$.

# Conclusions

We have presented the definition of an architecture for constructive geometric constraint solvers.

- The architecture is precisely and concisely defined.
- It is independent of any particular implementation of the functional units.
- It is well suited for interactive applications.
- The functional units are reusable to solve problems which are not geometric constraint solving problems but are related.

# Future work

Development of a common platform for researchers in geometric constraint solving.

This requires:

- The definition of data interchange standards for the data entities.
- The definition of a common set of geometric elements, constraints and geometric operations and its precise semantics.