DPLL and Proofs

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Logic and Algebra in Computer Science

Session 6

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Departament de Llenguatges i Sistemes Informatics

UNIVERSITAT POLITÈCNICA DE CATALUNYA

Overview of the session

- Proofs from CDCL SAT solvers
- Proofs from DPLL SAT solvers
- Some theoretical results

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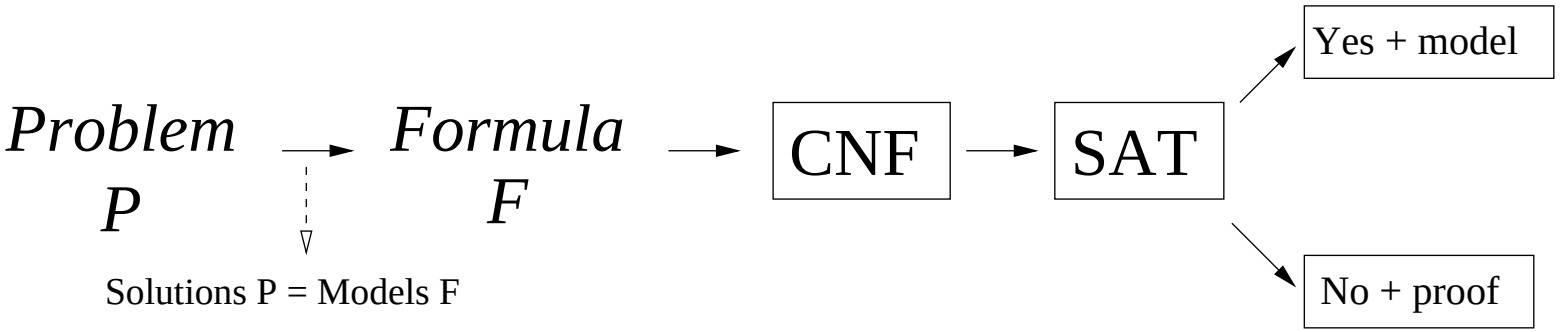
Problem solving with SAT



- This is the standard way of solving problems with SAT
- Remember: SAT box can be a DPLL-based SAT solver
  - Trivial to recover models
  - How to obtain proofs of unsatisfiability?

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$\emptyset \Longrightarrow$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$\emptyset \Longrightarrow 6^{\mathrm{d}} \Longrightarrow$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}} 2^{\mathrm{d}} \Longrightarrow$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}} 2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}} 2^{\mathsf{d}} \overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0

9 * 8 5 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}}2^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}}2^{\mathrm{d}}\overline{3}^{\mathrm{d}} \underset{p5}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}\overline{3}^{\mathrm{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4}\overline{1} \underset{c1}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4}\overline{1} \underset{c1}{\Longrightarrow} 6^{\mathsf{d}}4 \underset{p3}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4}\overline{1} \underset{c1}{\Longrightarrow} 6^{\mathsf{d}}4 \underset{p3}{\Longrightarrow} 6^{\mathsf{d}}42 \underset{p9}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
```

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The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathsf{d}}2^{\mathsf{d}}3\overline{41} \underset{c1}{\Longrightarrow} 6^{\mathsf{d}}4 \underset{p3}{\Longrightarrow} 6^{\mathsf{d}}42 \underset{p9}{\Longrightarrow} 6^{\mathsf{d}}423 \underset{c4}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4}\overline{1} \underset{c1}{\Longrightarrow} 6^{\mathsf{d}}4 \underset{p3}{\Longrightarrow} 6^{\mathsf{d}}42 \underset{p9}{\Longrightarrow} 6^{\mathsf{d}}423 \underset{c4}{\Longrightarrow} \overline{4} \underset{p6}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
11 * 4 9 3 0
```

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The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}} \Longrightarrow 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}} \underset{p5}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}\overline{3}^{\mathsf{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathsf{d}}2^{\mathsf{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathsf{d}}2^{\mathsf{d}}3\overline{41} \underset{c1}{\Longrightarrow} 6^{\mathsf{d}}4 \underset{p3}{\Longrightarrow} 6^{\mathsf{d}}42 \underset{p9}{\Longrightarrow} 6^{\mathsf{d}}423 \underset{c4}{\Longrightarrow} \overline{4} \underset{p6}{\Longrightarrow} \overline{42} \underset{p9}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
11 * 4 9 3 0
```

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The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}}2^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}}2^{\mathrm{d}}\overline{3}^{\mathrm{d}} \underset{p5}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}\overline{3}^{\mathrm{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathrm{d}}2^{\mathrm{d}}3\overline{4}\overline{1} \underset{c1}{\Longrightarrow} 6^{\mathrm{d}}4 \underset{p3}{\Longrightarrow} 6^{\mathrm{d}}42 \underset{p9}{\Longrightarrow} 6^{\mathrm{d}}423 \underset{c4}{\Longrightarrow} \overline{4} \underset{p6}{\Longrightarrow} \overline{4}2 \underset{p9}{\Longrightarrow} \overline{4}23 \underset{c7}{\Longrightarrow}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
11 * 4 9 3 0
```

The case of CDCL SAT solvers

$$\underbrace{1 \vee 4 \vee \overline{6}}_{1} \quad \underbrace{\overline{1} \vee 4}_{2} \quad \underbrace{2 \vee \overline{4}}_{3} \quad \underbrace{\overline{2} \vee \overline{3} \vee \overline{4}}_{4} \quad \underbrace{\overline{2} \vee 3 \vee 5}_{5} \quad \underbrace{2 \vee 4}_{6} \quad \underbrace{\overline{3} \vee 4}_{7} \quad \underbrace{\overline{2} \vee 3 \vee \overline{5}}_{8}$$

$$\emptyset \Longrightarrow 6^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}}2^{\mathrm{d}} \Longrightarrow 6^{\mathrm{d}}2^{\mathrm{d}}\overline{3}^{\mathrm{d}} \underset{p5}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}\overline{3}^{\mathrm{d}}5 \underset{c8}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}3 \underset{p4}{\Longrightarrow} 6^{\mathrm{d}}2^{\mathrm{d}}3\overline{4} \underset{p2}{\Longrightarrow}$$

$$6^{\mathrm{d}}2^{\mathrm{d}}3\overline{41} \underset{c1}{\Longrightarrow} 6^{\mathrm{d}}4 \underset{p3}{\Longrightarrow} 6^{\mathrm{d}}42 \underset{p9}{\Longrightarrow} 6^{\mathrm{d}}423 \underset{c4}{\Longrightarrow} \overline{4} \underset{p6}{\Longrightarrow} \overline{4}2 \underset{p9}{\Longrightarrow} \overline{4}23 \underset{c7}{\Longrightarrow} \mathit{fail}$$

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
```

```
9 * 8 5 0
10 * 1 2 0
11 * 4 9 3 0
12 * 7 9 6 11 0
```

Trace files

Trace file:

```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
9 * 8 5 0
10 * 1 2 0
11 * 4 9 3 0
12 * 7 9 6 11 0
```

- Each line starts with clause identifier
- If * follows, it is a lemma
  (generated by resolution on the following clause ids)
- Otherwise, it is an input clause
- Last lemma should generate the empty clause

Processing Trace Files - DFS

Trace file:

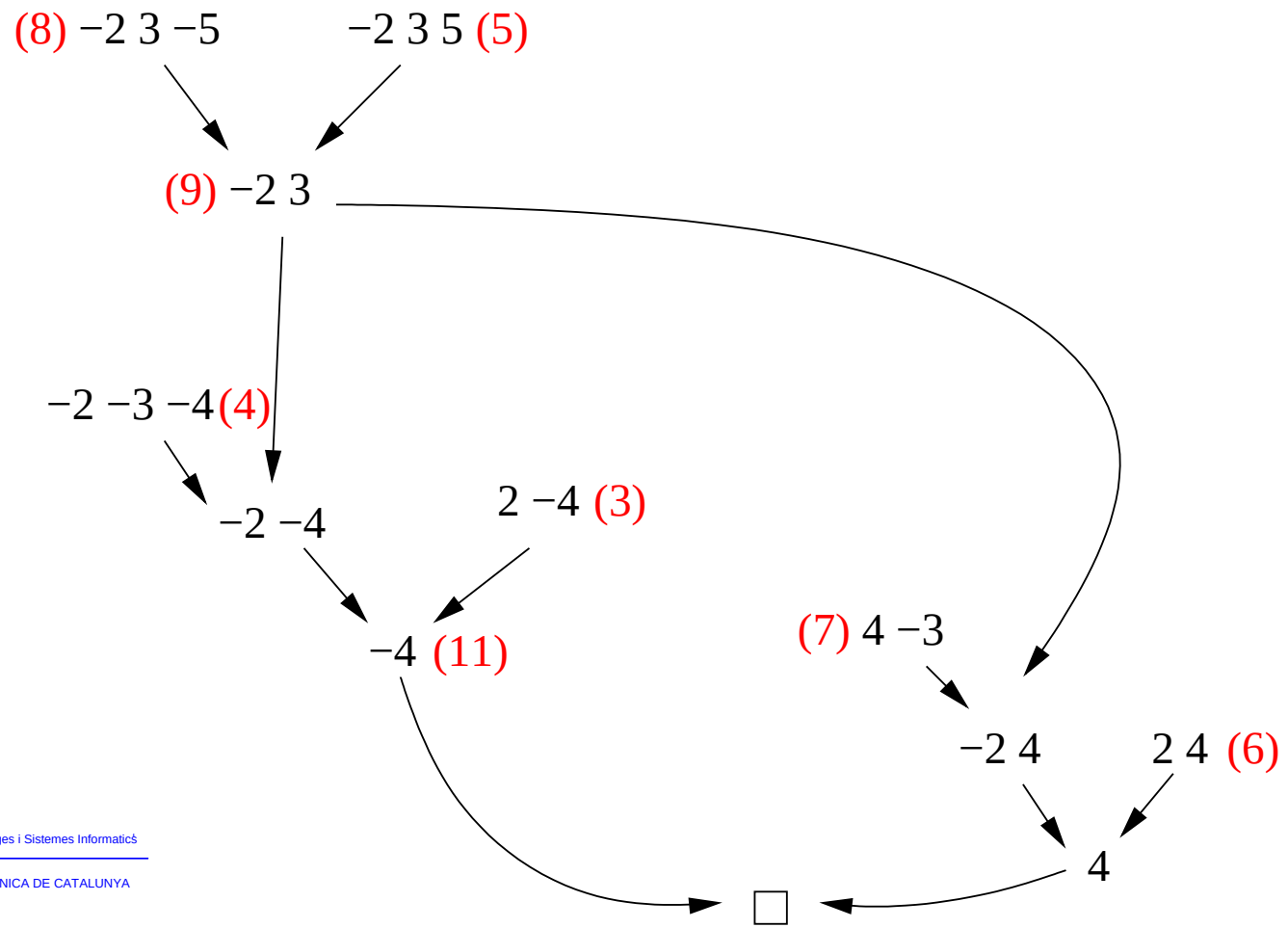
```
1 1 4 -6 0 0
2 -1 4 0 0
3 2 -4 0 0
4 -2 -3 -4 0 0
5 -2 3 5 0 0
6 2 4 0 0
7 -3 4 0 0
8 -2 3 -5 0 0
9 * 8 5 0
10 * 1 2 0
11 * 4 9 3 0
12 * 7 9 6 11 0
```

- Start with last clause
- Check whether resolution is possible with the clause ids
- If clause id not input clause, recursively generate clause
- For efficiency, all trace into memory: NOT FEASIBLE

Processing Trace Files - DFS (2)

Trace file:

```
9 * 8 5 0                    11 * 4 9 3 0
10 * 1 2 0                   12 * 7 9 6 11 0
```



Processing Trace Files - DFS(3)

- We can easily identify an unsatisfiable core:
  Subset of input clauses which is unsatisfiable
- Unsatisfiable cores have lots of applications:
  - CP problems: indentify unsatisfiable sets of constraints
  - Abstraction refinement in verification
- Cores obtained are not guaranteed to be minimal
- Iterating the process may reduce the core size
- How to avoid DFS memory problems? BFS

Processing Trace Files - BFS

- Traverse learned clauses in the generated order
- Generated all learned clauses
- When reaching empty clause all lemmas are available
- **PROBLEM:** too many lemmas in memory
- **SOLUTION:**
  - first pass counts number of times each lemma is used
  - in second pass lemmas no longer used are removed
  - same memory as used by SAT solver

Overview of the session

- Proofs from CDCL SAT solvers
- Proofs from DPLL SAT solvers
- Some theoretical results

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The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \quad \underbrace{1 \vee \overline{2} \vee 3}_{2} \quad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \quad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \quad \underbrace{\overline{3} \vee 4}_{5} \quad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \quad \underbrace{1 \vee \overline{2} \vee 3}_{2} \quad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \quad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \quad \underbrace{\overline{3} \vee 4}_{5} \quad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \quad \underbrace{1 \vee \overline{2} \vee 3}_{2} \quad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \quad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \quad \underbrace{\overline{3} \vee 4}_{5} \quad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}} 2^{\mathsf{d}} \underset{p4}{\Longrightarrow}$$

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The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \quad \underbrace{1 \vee \overline{2} \vee 3}_{2} \quad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \quad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \quad \underbrace{\overline{3} \vee 4}_{5} \quad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} \overline{1}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} \overline{1}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} \overline{12} \underset{p1}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} \overline{1}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} \overline{12} \underset{p1}{\Longrightarrow} \overline{12}3 \underset{p5}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \qquad \underbrace{1 \vee \overline{2} \vee 3}_{2} \qquad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \qquad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \qquad \underbrace{\overline{3} \vee 4}_{5} \qquad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} \overline{1}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} \overline{12} \underset{p1}{\Longrightarrow} \overline{12}3 \underset{p5}{\Longrightarrow} \overline{12}34 \underset{c6}{\Longrightarrow}$$

The case of Basic DPLL

$$\underbrace{1 \vee 2 \vee 3}_{1} \quad \underbrace{1 \vee \overline{2} \vee 3}_{2} \quad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \quad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \quad \underbrace{\overline{3} \vee 4}_{5} \quad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} \overline{1}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} \overline{12} \underset{p1}{\Longrightarrow} \overline{12}3 \underset{p5}{\Longrightarrow} \overline{12}34 \underset{c6}{\Longrightarrow}$$

fail

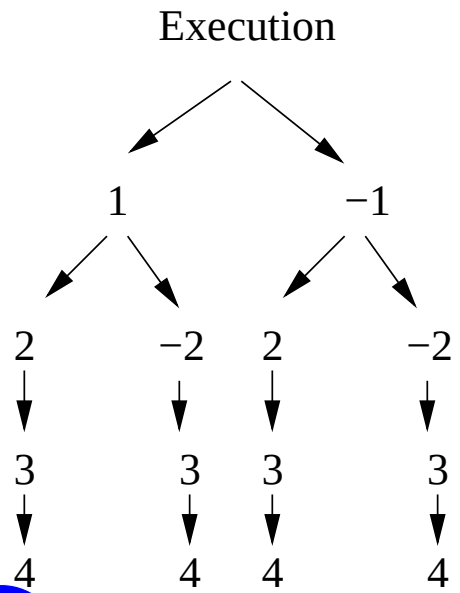
The case of Basic DPLL

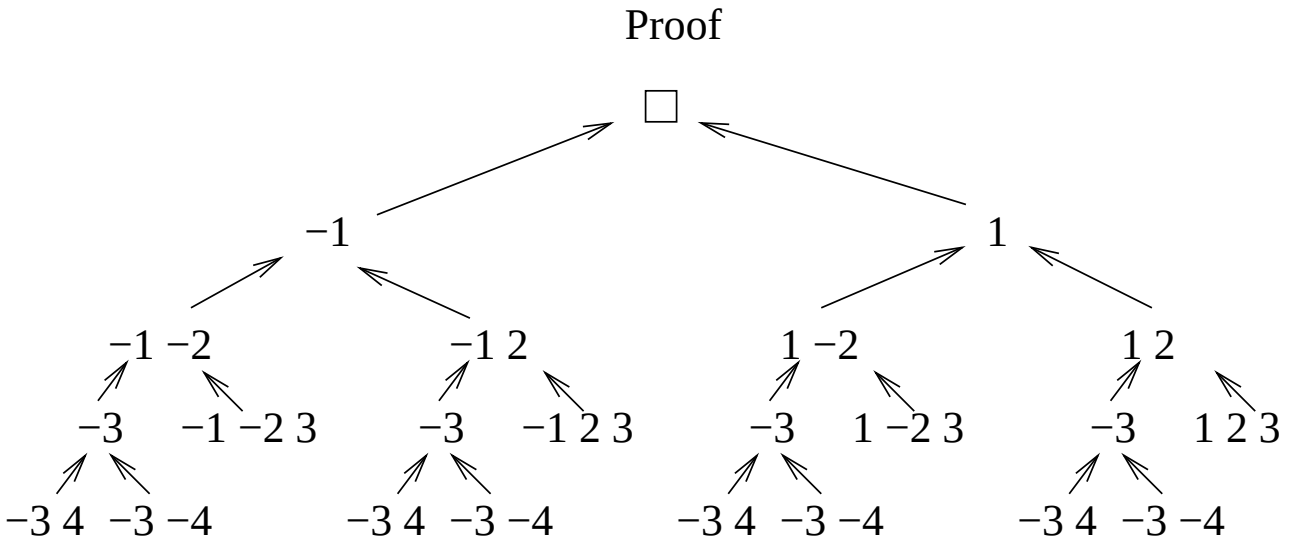
$$\underbrace{1 \vee 2 \vee 3}_{1} \quad \underbrace{1 \vee \overline{2} \vee 3}_{2} \quad \underbrace{\overline{1} \vee 2 \vee 3}_{3} \quad \underbrace{\overline{1} \vee \overline{2} \vee 3}_{4} \quad \underbrace{\overline{3} \vee 4}_{5} \quad \underbrace{\overline{3} \vee \overline{4}}_{6}$$

$$\emptyset \Longrightarrow 1^{\mathsf{d}} \Longrightarrow 1^{\mathsf{d}}2^{\mathsf{d}} \underset{p4}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} 1^{\mathsf{d}}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} 1^{\mathsf{d}}\overline{2} \underset{p3}{\Longrightarrow} 1^{\mathsf{d}}\overline{2}3 \underset{p5}{\Longrightarrow}$$

$$1^{\mathsf{d}}\overline{2}34 \underset{c6}{\Longrightarrow} \overline{1} \Longrightarrow \overline{1}2^{\mathsf{d}} \underset{p2}{\Longrightarrow} \overline{1}2^{\mathsf{d}}3 \underset{p5}{\Longrightarrow} \overline{1}2^{\mathsf{d}}34 \underset{c6}{\Longrightarrow} \overline{12} \underset{p1}{\Longrightarrow} \overline{12}3 \underset{p5}{\Longrightarrow} \overline{12}34 \underset{c6}{\Longrightarrow}$$

fail





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Overview of the session

- Proofs from CDCL SAT solvers
- Proofs from DPLL SAT solvers
- Some theoretical results

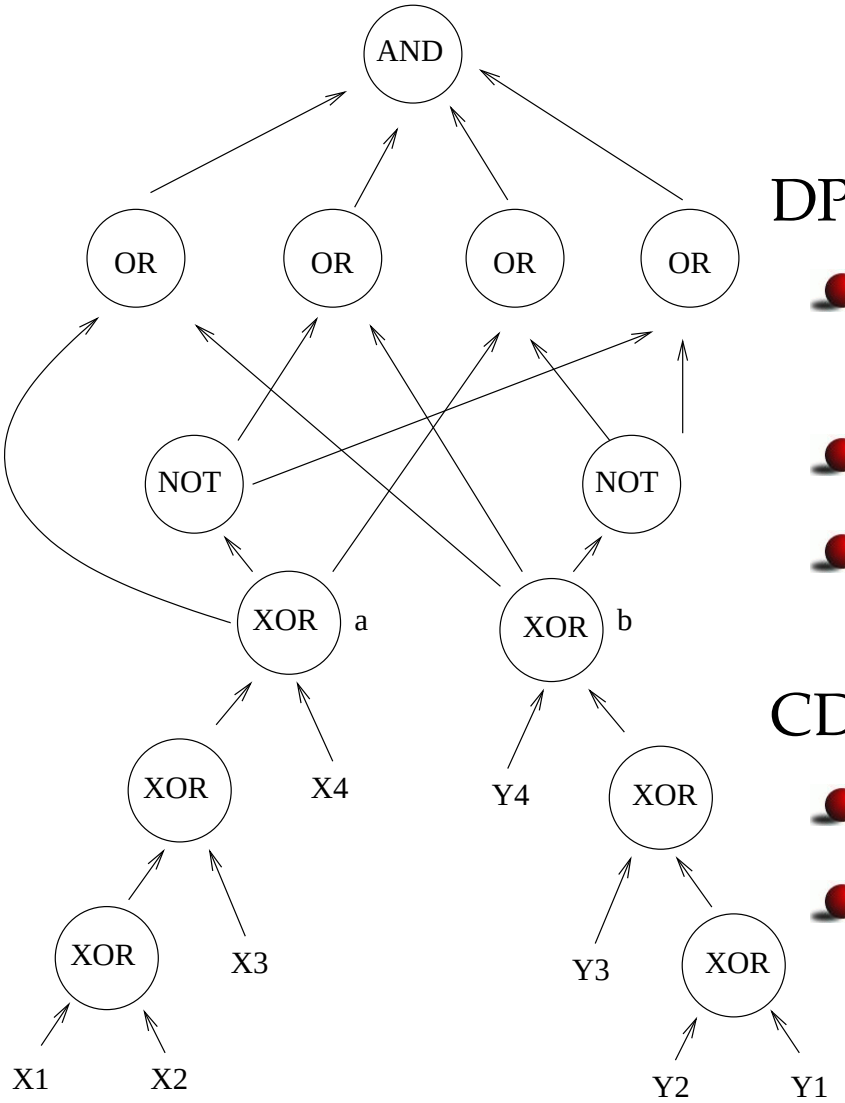
Resolution variants and DPLL variants

- Basic DPLL:
  - Every run of basic DPLL visiting N nodes gives a tree-like resolution proof of size polynomial in N
  - Every tree-like resolution proof of size S corresponds to a DPLL run visiting a number of nodes polynomial in S
  - We say that DPLL and tree-like resolution are polynomially equivalent
- CDCL:
  - Every run of CDCL visiting N nodes gives a general proof by resolution of size polynomial in N
  - Every general resolution proof of size S corresponds to a CDCL run visiting a number of nodes polynomial in S
  - We say that CDCL and general resolution are polynomially equivalent

Resolution variants and DPLL variants (2)

- Every tree-like proof is a general proof
- There are formulas whose tree-like refutations are exponentially larger than the shortest general refutation
- Hence, on those formulas all runs of DPLL are exponentially longer that the shortest CDCL run
- Note that nothing is said about how easy it is to find the shortest CDCL run

Branching on inputs



DPLL:

- Restrict DPLL to branch on inputs
- Any run proportional to # inputs
- But branching on a, b has runs of cnt. time

CDCL:

- Restrict CDCL to branch on input
- There are runs of cnt. time

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Branching on inputs (2)

- In practice, restrict branching to inputs not recommended
- Some theoretical results:
  - *DPLL* is exponentially weaker than *CDCL*
  - $DPLL_{inputs}$ cannot polynomially simulate *DPLL*
  - *DPLL* and $CDCL_{inputs}$ are incomparable
  - $CDCL_{inputs}$ cannot polynomially simulate *CDCL*

Bibliography - Some further reading

- Lintao Zhang, Sharad Malik. *Validating SAT Solvers Using an Independent Resolution-Based Checker: Practical Implementations and Other Applications.* DATE 2003: 10880-10885

- Paul Beame, Henry A. Kautz, Ashish Sabharwal. *Understanding the Power of Clause Learning*. IJCAI 2003: 1194-1201

- Matti Järvisalo, Tommi A. Junttila. *Limitations of restricted branching in clause learning. Constraints 14(3): 325-356 (2009)*