Structural Facilitation and Structural Inhibition

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Abstract

The paper addresses constraints on long-distance extraction in categorial grammar, involving formulation and application of logical extensions of Lambek calculus. Structural facilitation, i.e. controlled import of structural properties from higher in the hierarchy of substructural logics, is complemented by a proposal for structural inhibition: controlled import of structural properties from lower in the hierarchy. A treatment is developed which includes island constraints, licensing of subject extraction, 'assure'-type "extraction only" valencies, 'whom'-binding of downstairs but not upstairs nominative positions, and variation in the penetrative power of fillers.

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Coordination of non-constituents, prosodic phrasing, and incremental interpretation have been taken to motivate a dissolution of constituent structure in categorial grammar (Steedman 1987; Dowty 1987). The associative Lambek calculus instantiates the most extreme position in this respect: it recognises no constituent structure at all. And at the same time as being totally undiscriminating with respect to dominance structure, it is totally rigid with respect to linear ordering. In these senses associative Lambek calculus is linguistically both too strong and too weak, overgenerating as regards domain constraints, and undergenerating as regards linear flexibility.

Associative Lambek calculus occupies a position in a substructural hierarchy of logics between the stronger linear logic (which recognises no linear ordering) and the weaker non-associative Lambek calculus (which recognises binary dominance structure). The limitation with respect to order variation has been addressed by adding structural operators effecting structural facilitation (Morrill et al. 1990; Barry et al. 1991), in the spirit of linear logic (Girard 1987). This constitutes controlled import of structural properties from stronger logics in the substructural hierarchy. The present article develops the proposal of Morrill (1992) to move in the opposite direction also, using operators for structural inhibition: controlled import of structural properties from a logic weaker in the substructural hierarchy (the non-associative Lambek calculus). An embedding translation is conjectured. Structural facilitation and inhibition are put to work together in the characterisation of islands to extraction (Complex Noun Phrase Constraint, Subject Condition, subject of possessive, and Coordinate Structure Constraint), and in the characterisation of differential penetrability.

1 Categorial Grammar

This paper concerns itself with constraints on long-distance extraction in categorial grammar in the "logical" tradition. Such a tradition can be contrasted with the "unificatory" one (cf. HPSG; Pollard and Sag 1987, 1993) where unbounded dependencies are mediated by feature percolation, and the "combinatory" one, where they are mediated by non-logical rules, i.e. rules which are not the theorems of an interpretation of categorial types. The logical approach (Moortgat 1988, van Benthem 1991, Morrill 1992) by contrast aims to generate under rules which are the logical validities according to the meaning of categorial connectives.

The core categorial calculi on this design (based on interpretation by residuation in semigroups and groupoids, cf. e.g. Lambek 1987) are the associative and non-associative Lambek calculi (Lambek 1958, 1961). The non-associative system **NL** projects a hierarchical structure which blocks non-local dependencies. But the associative system **L** delivers unbounded dependencies under assignment of fronted elements such as relative pronouns to types of the form R/(S/N). There is however only scope to allow extraction from left or right *frontier* positions; furthermore, the associative Lambek calculus offers no form of *constraint* on extraction, e.g. domains such as complex noun phrases and coordinate structures cannot be specified as islands. The present proposals aim to redress this under- and over-generation.

With respect to the undergeneration, two approaches facilitating medial extraction have been forwarded within the logical tradition: discontinuity operators (Moortgat 1988) under which a relative pronoun may be a higher-order functor over a "wrapping" functor: $\mathrm{R}/(\mathrm{S}{\uparrow}\mathrm{N})$, and structural operators which allow assignment as a functor over a functor over e.g. commutable N: $\mathrm{R}/(\mathrm{S}/\triangle\mathrm{N})$. We will argue that the latter approach probably offers most prospects for treatments including domains which are semi-islandlike.

One way of increasing the discriminatory power of basic Lambek calculus is to add universal modality (Morrill 1989, 1990). Where certain functors are of category $B/\Box A$, inducing a modal domain on their argument, a higher order functor $C/(D/E)$ can only exercise a dependency into the modal domain if E itself is modalised, i.e. is of the form $\Box E'$. For such effects to be made to work however each lexical category needs to be modalised. These circumstances arise naturally in the use of semantically active universal modality for intensionality, meaning that some constraints can be formulated with respect to intensional domains (e.g. tensed S constraints). Other constraints cannot be so expressed however, and we pursue here another strategy of structural inhibition.

This strategy, employed with respect to the overgeneration, is one dual to structural facilitation: it comprises controlled import into a logic **L** higher in the structural hierarchy of structural properties (non-associativitity) from the logic **NL** lower in the structural hierarchy. This was done in Morrill (1992) by means of categorial types formed under unary "bracket" ($[\,]$) and "antibracket") ($[\,]^{-1}$) operators interpreted with respect to a unary bracketing operation and its inverse. For this a translation embedding **NL** into $\mathbf{L}+\{[\,],[\,]^{-1}\}$, that is one in the reverse of the usual direction for substructural embedding, was conjectured. In relation to linguistic matters, the domain operators can be used to mark complex noun phrases, coordinate structures, sentencial subjects, and subjects of possessives as islands. In the version promoted here, rather than having *two* operators, defined with respect to *an operation and its inverse*, we have *one* "bar" operator defined with respect to an operation which is *self-inverse*. This depopulates the algebra of interpretation of (apparently) linguistically inapplicable stacked bracketings and antibracketings while maintaining the treatment of constraints.

2 The Structural Hierarchy: Gentzen Sequent Calculus

We assume a set $\mathcal{F}$ of syntactic type (or: "category") formulas freely generated from a set $\mathcal{A}$ of atoms thus:

(1) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F}$

A sequent of the non-associative Lambek calculus **NL**, written $\Gamma \Rightarrow A$, comprises a succedent type A and an antecedent configuration Γ, where the configurations $\mathcal{O}$ are freely generated from formulas $\mathcal{F}$ as shown in (2).

(2) $\mathcal{O} = \mathcal{F} \mid (\mathcal{O}, \mathcal{O})$

The theorems of **NL** are those sequents that can be generated from the axiomatic rule id, and the rules Cut, \L, \R, /L, /R, $\bullet$L and $\bullet$R. The notation $\Gamma[\Delta]$ here refers to a configuration Γ with a distinguished subconfiguration Δ.

This sequent calculus has the property of Cut-elimination (Lambek 1961), so that every theorem has a Cut-free proof.

(3) a.
$$\frac{}{A \Rightarrow A}\text{id} \qquad \frac{\Gamma \Rightarrow A \quad \Delta[A] \Rightarrow B}{\Delta[\Gamma] \Rightarrow B}\text{Cut}$$

b.
$$\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[(\Gamma, A\backslash B)] \Rightarrow C}\backslash\text{L} \qquad \frac{(A, \Gamma) \Rightarrow B}{\Gamma \Rightarrow A\backslash B}\backslash\text{R}$$

c.
$$\frac{\Gamma \Rightarrow A \quad \Delta[B] \Rightarrow C}{\Delta[(B/A, \Gamma)] \Rightarrow C}/\text{L} \qquad \frac{(\Gamma, A) \Rightarrow B}{\Gamma \Rightarrow B/A}/\text{R}$$

d.
$$\frac{\Gamma[(A, B)] \Rightarrow C}{\Gamma[A\bullet B] \Rightarrow C}\bullet\text{L} \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{(\Gamma, \Delta) \Rightarrow A\bullet B}\bullet\text{R}$$

By way of example, there is the following derivation of lifting, $\mathrm{A} \Rightarrow \mathrm{B}/(\mathrm{A}\backslash\mathrm{B})$:

(4)
$$\frac{\dfrac{\mathrm{A} \Rightarrow \mathrm{A} \quad \mathrm{B} \Rightarrow \mathrm{B}}{(\mathrm{A}, \mathrm{A}\backslash\mathrm{B}) \Rightarrow \mathrm{B}}\backslash\text{L}}{\mathrm{A} \Rightarrow \mathrm{B}/(\mathrm{A}\backslash\mathrm{B})}/\text{R}$$

A Gentzen-style calculus for the associative Lambek calculus **L** can be obtained by adding a structural rule of associativity to **NL**:

(5)
$$\frac{\Gamma[((\Delta_1, \Delta_2), \Delta_3)] \Rightarrow A}{\Gamma[(\Delta_1, (\Delta_2, \Delta_3))] \Rightarrow A}\text{A}$$

(Double lines indicate that rules are valid reading both up and down.) Then, for example division, $\mathrm{B}/\mathrm{A} \Rightarrow (\mathrm{B}/\mathrm{C})/(\mathrm{A}/\mathrm{C})$, which is not derivable in **NL**, can be obtained thus:

(6)
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{C} \Rightarrow \text{C} \qquad \text{A} \Rightarrow \text{A}}{(\text{A/C, C}) \Rightarrow \text{A}}\text{/L} \qquad \text{B} \Rightarrow \text{B}}{(\text{B/A, (A/C, C)}) \Rightarrow \text{B}}\text{/L}}{((\text{B/A, A/C}), \text{C}) \Rightarrow \text{B}}\text{A}}{(\text{B/A, A/C}) \Rightarrow \text{B/C}}\text{/R}}{\text{B/A} \Rightarrow (\text{B/C})/(\text{A/C})}\text{/R}$$

The associative Lambek calculus may also be presented with associativity made implicit. Configurations are generated under an n+1-ary constructor $\cdot, \ldots, \cdot$:

(7) $\mathcal{O} = \mathcal{F}, \ldots, \mathcal{F}$

Then the following rules also enjoy Cut-elimination (Lambek 1958):

(8) a. $\dfrac{}{A \Rightarrow A}\text{id} \qquad \dfrac{\Gamma \Rightarrow A \qquad \Delta_1, A, \Delta_2 \Rightarrow B}{\Delta_1, \Gamma, \Delta_2 \Rightarrow B}\text{Cut}$

b. $\dfrac{\Gamma \Rightarrow A \qquad \Delta_1, B, \Delta_2 \Rightarrow C}{\Delta_1, \Gamma, A\backslash B, \Delta_2 \Rightarrow C}\backslash\text{L} \qquad \dfrac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A\backslash B}\backslash\text{R}$

c. $\dfrac{\Gamma \Rightarrow A \qquad \Delta_1, B, \Delta_2 \Rightarrow C}{\Delta_1, B/A, \Gamma, \Delta_2 \Rightarrow C}/\text{L} \qquad \dfrac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow B/A}/\text{R}$

d. $\dfrac{\Gamma_1, A, B, \Gamma_2 \Rightarrow C}{\Gamma_1, A\bullet B, \Gamma_2 \Rightarrow C}\bullet\text{L} \qquad \dfrac{\Gamma \Rightarrow A \qquad \Delta \Rightarrow B}{\Gamma, \Delta \Rightarrow A\bullet B}\bullet\text{R}$

In this format the proof (9) of division contains no explicit association step.

(9)
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{C} \Rightarrow \mathrm{C} \quad \mathrm{A} \Rightarrow \mathrm{A}}{\mathrm{A/C,\ C} \Rightarrow \mathrm{A}}/\mathrm{L} \quad \mathrm{B} \Rightarrow \mathrm{B}}{\mathrm{B/A,\ A/C,\ C} \Rightarrow \mathrm{B}}/\mathrm{L}}{\mathrm{B/A,\ A/C} \Rightarrow \mathrm{B/C}}/\mathrm{R}}{\mathrm{B/A} \Rightarrow \mathrm{(B/C)/(A/C)}}/\mathrm{R}$$

When we add also a structural rule of permutation (10) (or: exchange, or: commutativity) the result is the so-called Lambek-van Benthem calculus **LL**.

(10)
$$\dfrac{\Gamma_1, A, B, \Gamma_2 \Rightarrow C}{\Gamma_1, B, A, \Gamma_2 \Rightarrow C}\mathrm{P}$$

The distinction in directionality of implications collapses; for example we have (11).

(11)
$$\dfrac{\dfrac{\dfrac{\mathrm{A} \Rightarrow \mathrm{A} \quad \mathrm{B} \Rightarrow \mathrm{B}}{\mathrm{A,\ A\backslash B} \Rightarrow \mathrm{B}}\backslash\mathrm{L}}{\mathrm{A\backslash B,\ A} \Rightarrow \mathrm{B}}\mathrm{P}}{\mathrm{A\backslash B} \Rightarrow \mathrm{B/A}}/\mathrm{R}$$

The non-directional implication and the product correspond to the linear implication and tensor product of intuitionistic linear logic, these comprising what is referred to as the multiplicative fragment. There is the slight difference that linear logic, but not the categorial calculi, may have empty antecedents.

The variations we have seen exist within what we may call the sublinear hierarchy. The structural hierarchy continues upwards to relevance logic (Anderson and Belnap 1975) and intuitionistic logic with addition of contraction (12a) and weakening (or: monotonicity) (12b).

(12) a. $$\frac{\Gamma_1, A, A, \Gamma_2 \Rightarrow B}{\Gamma_1, A, \Gamma_2 \Rightarrow B}\mathrm{C}$$ b. $$\frac{\Gamma_1, \Gamma_2 \Rightarrow B}{\Gamma_1, A, \Gamma_2 \Rightarrow B}\mathrm{C}$$

In the following section we turn to a perspective on the hierarchy from the point of view of interpretation.

3 Model Theory

3.1 Multiplicative Operators and Groupoid Algebras: interpretive perspective on the structural hierarchy

We get a tour of the substructural landscape, i.e. the space of logics obtained by dropping structural rules (Došen and Schroeder-Heister 1993), by considering interpretation with respect to various model structures, starting with a groupoid algebra $\langle L, +\rangle$ which is simply a set L closed under a binary operation $+$. An interpretation is a mapping D of formulas into subsets of L such that (cf. e.g. Lambek 1988):

(13)
$$\begin{array}{rcl} D(A\bullet B) & = & \{s_1+s_2 | s_1 \in D(A) \wedge s_2 \in D(B)\} \\ D(A\backslash B) & = & \{s | \forall s' \in D(A), s'+s \in D(B)\} \\ D(B/A) & = & \{s | \forall s' \in D(A), s+s' \in D(B)\} \end{array}$$

We refer to this scheme as interpretation by *residuation*. It defines a consequence relation $\models$ between formulas thus:

(14) $A \models B$ iff in all interpretations $D(A) \subseteq D(B)$

This gives us the theory of the non-associative Lambek calculus **NL**. Keeping the interpretation clauses and varying the algebra gives us a range of substructural logics: non-associative Lambek calculus, associative Lambek calculus, linear logic, relevance logic. If we impose the condition of associativity (15) on the algebra of interpretation, we are dealing with semigroup algebras $\langle L, +\rangle$.

(15) $s_1+(s_2+s_3) = (s_1+s_2)+s_3$

This gives us associative Lambek calculus **L**, a version of non-commutative linear logic. If we further impose the condition of commutativity (16) we have commutative (or: Abelian) semigroup algebras $\langle L, +\rangle$.

(16) $s_1{+}s_2 = s_2{+}s_1$

This gives the Lambek-van Benthem calculus, a version of (the multiplicative fragment of) linear logic. And if we further impose the condition of idempotency (17) we have semi-lattice algebras $\langle L, +\rangle$.

(17) $s{+}s = s$

This gives a version of relevance logic.[1] The groupoid interpretation characterises the prosodic content to categorial classification by types. But there is also a semantic side given by the Curry-Howard "propositions as types" correspondence between proofs and lambda terms. When these are put together we obtain a categorial logic of signs (prosodic/semantic associations) which is the framework for linguistic application.

3.2 Type-logical Semantic Interpretation

A set $\mathcal{T}$ of semantic type indices is freely generated from a set $\mathcal{D}$ of basic semantic type indices thus:

(18) $\mathcal{T} = \mathcal{D} \mid \mathcal{T} \to \mathcal{T} \mid \mathcal{T} \times \mathcal{T}$

Semantic interpretation of category formulas is in a many-sorted algebra with sorts $\{\tau | \tau \in \mathcal{T}\}$:

(19) $\langle \{D_\tau\}_{\tau\in\mathcal{T}}, \{(\cdot\ \cdot)_{\tau_1,\tau_2}\}_{\tau_1,\tau_2\in\mathcal{T}}, \{(\cdot,\cdot)_{\tau_1,\tau_2}\}_{\tau_1,\tau_2\in\mathcal{T}}\rangle$

Here, $\{D_\tau\}_{\tau\in\mathcal{T}}$ is a family of sets (semantic domains), a *frame*, such that $D_{\tau_1\to\tau_2}$ is the set of all (set-theoretic) functions from D_{τ_1} to D_{τ_2} (function space) and $D_{\tau_1\times\tau_2}$ is the set of all ordered pairs of objects from D_{τ_1} and D_{τ_2} respectively (Cartesian product, or: cross product). For each $\tau_1, \tau_2 \in \mathcal{T}$, $(\cdot\ \cdot)_{\tau_1,\tau_2}$ is application of the function that is its first operand, of type $\tau_1{\to}\tau_2$, to the argument that is its second operand, of type τ_1, yielding a value of

[1] We would arrive at intuitionistic logic by constraining D to satisfy persistence (or: heridity), $s{+}s' \in D(A)$ if $s \in D(A)$.

type τ_2; and $(\cdot,\cdot)_{\tau_1,\tau_2}$ is ordered pairing of its first operand, of type τ_1, to its second operand, of type τ_2, yielding an object of type $\tau_1{\times}\tau_2$.

A type map is a function T from category formulas to semantic types such that

(20) $T(A\backslash B) = T(B/A) = T(A) \to T(B)$
$T(A{\bullet}B) = T(A) \times T(B)$

Working in the two dimensions, prosodic and semantic, to obtain a suitable logic of signs, each formula A has an interpretation $D(A)$ which is a set of pairs of prosodic objects from L and semantic objects from $T(A)$ (cf. Morrill 1992a):

(21)
$$\begin{array}{lcl} D(A{\bullet}B) & = & \{\langle s_1{+}s_2, \langle m_1, m_2\rangle\rangle | \langle s_1, m_1\rangle \in D(A) \wedge \langle s_2, m_2\rangle \in D(B)\} \\ D(A\backslash B) & = & \{\langle s, m\rangle | \forall \langle s', m'\rangle \in D(A), \langle s'{+}s, m(m')\rangle \in D(B)\} \\ D(B/A) & = & \{\langle s, m\rangle | \forall \langle s', m'\rangle \in D(A), \langle s{+}s', m(m')\rangle \in D(B)\} \end{array}$$

4 Fitch-Style Proof Theory

We use labelled deductive systems for presentation of proof calculi (Gabbay 1991; see Moortgat 1991 for categorial application). In addition to a language of formulas interpreted as sets of objects, there is defined some language of terms (labels) interpreted as such objects. Statements of the form l: A assert that the object represented by term l belongs to type A. Our labels will actually be pairings of prosodic and semantic terms, and we will present a Fitch-style natural deduction for the type assignment system (Morrill 1993).

4.1 Prosodic and Semantic terms

For groupoid models, a set $\mathcal{P}$ of prosodic terms is freely generated from a set $\mathcal{K}$ of prosodic constants and a denumerably infinite set $\mathcal{U}$ of prosodic variables thus:

(22) $\mathcal{P} = \mathcal{U} \mid \mathcal{K} \mid \mathcal{P}{+}\mathcal{P}$

Each prosodic term α has an interpretation as an object in a groupoid algebraic model structure, given in the obvious way.

To include the semantic side, typed lambda terms are defined and interpreted as usual. Starting from a set $\mathcal{C}_\tau$ of constants for each type τ and a denumerably infinite set $\mathcal{V}_\tau$ of variables for each type τ, the set $\mathcal{S}_\tau$ of typed semantic terms for each type τ is freely generated thus:

(23)
$$\begin{array}{lll} \mathcal{S}_\tau & = & \mathcal{C}_\tau \mid \mathcal{V}_\tau \mid (\mathcal{S}_{\tau'\to\tau}\ \mathcal{S}_{\tau'}) \mid \pi_1\mathcal{S}_{\tau\times\tau'} \mid \pi_2\mathcal{S}_{\tau'\times\tau} \\ \mathcal{S}_{\tau'\to\tau} & = & \lambda\mathcal{V}_{\tau'}\mathcal{S}_\tau \\ \mathcal{S}_{\tau\times\tau'} & = & (\mathcal{S}_\tau, \mathcal{S}_{\tau'}) \end{array}$$

For labelled Fitch-style natural deduction for the non-associative Lambek calculus **NL** there are the following rules of lexical insertion, subderivation hypothesis, and label manipulation.

(24) a. $n.$ $\alpha - \phi$: A for any lexical entry

b.
$$\begin{array}{r|ll} n. & a_1 - x_1\colon A_1 & \mathrm{H} \\ & \vdots & \vdots \\ n+m. & \underline{a_m - x_m\colon A_n} & \mathrm{H} \end{array}$$

c.
$$\begin{array}{rll} n. & \alpha - \phi\colon A & \\ & \alpha' - \phi'\colon A & = n, \text{if } \alpha = \alpha' \ \&\ \phi = \phi' \end{array}$$

Then there are logical rules of elimination and introduction for each operator:

(25) a.
$$\begin{array}{rll} n. & \alpha - \phi\colon A & \\ m. & \gamma - \chi\colon A\backslash B & \\ & (\alpha{+}\gamma) - (\chi\ \phi)\colon B & \mathrm{E}\backslash\ n, m \end{array}$$

b.
$$\begin{array}{rll} n. & \vert\ \underline{a - x\colon A} & \mathrm{H} \\ m. & \vert\ (a{+}\gamma) - \psi\colon B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x\psi\colon A\backslash B & \mathrm{I}\backslash\ n, m \end{array}$$

(26) a.
$$\begin{array}{rll} n. & \alpha - \phi\colon A & \\ m. & \gamma - \chi\colon B/A & \\ & (\gamma{+}\alpha) - (\chi\ \phi)\colon B & \mathrm{E}/\ n, m \end{array}$$

b. $n.$ $a - x\colon A$ H
$m.$ $(\gamma{+}a) - \psi\colon B$ unique a as indicated
$\gamma - \lambda x\psi\colon B/A$ I/ n, m

(27) a. $n.$ $\gamma - \chi\colon A{\bullet}B$
$m.$ $a - x\colon A$ H
$m+1.$ $b - y\colon B$ H
$p.$ $\delta[(a{+}b)] - \omega[x, y]\colon D$ unique a and b as indicated
$\delta[\gamma] - \omega[\pi_1\chi, \pi_2\chi]\colon D$ E$\bullet$ $n, m, m+1, p$

b. $n.$ $\alpha - \phi\colon A$
$m.$ $\beta - \psi\colon B$
$(\alpha{+}\beta) - (\phi, \psi)\colon A{\bullet}B$ I$\bullet$ n, m

For the Fitch-style presentation of the associative Lambek calculus **L** we may add a prosodic equation:

(28) $(\alpha_1{+}\alpha_2){+}\alpha_3 = \alpha_1{+}(\alpha_2{+}\alpha_3)$

Alternatively, the associative Lambek calculus can be given by representing associative adjunction of n elements by an n-ary constructor: $\cdot{+}\ldots{+}\cdot$.

Correspondingly, for the Lambek/van Benthem calculus we may add the prosodic equation (29):

(29) $\alpha{+}\beta = \beta{+}\alpha$

We shall see examples of derivations in the course of linguistic explication.

5 Linguistic Application

5.1 Left extraction

The facility of hypothetical reasoning of the associative Lambek calculus allows it to provide a rudimentary characterisation of long-distance dependency constructions such as relativisation and topicalisation in which a fronted element binds a "gap" at the periphery of an otherwise complete sentence; the incomplete sentence is analysed as S/N:

(30) a. (the book) which$_i$ John talked about e_i
b. Mozart$_i$ John talked about e_i

1.	*which* $-$ $\lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]$: (CN\CN)/(S/N)	
2.	*John* $-$ **j**: N	
3.	*talked* $-$ **talk**: (N\S)/PP	
4.	*about* $-$ **about**: PP/N	
5.	*a* $-$ x: N	H
6.	*about+a* $-$ (**about** x): PP	4, 5 E/
7.	*talked+about+a* $-$ (**talk** (**about** x)): N\S	3, 6 E/
8.	*John+talked+about+a* $-$ ((**talk** (**about** x)) **j**): S	2, 7 E\
9.	*John+talked+about* $-$ λx((**talk** (**about** x)) **j**): S/N	5, 8 I/
10.	*which+John+talked+about* $-$	
	($\lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]$ λx((**talk** (**about** x)) **j**)): CN\CN	1, 9 E/
11.	*which+John+talked+about* $-$	
	$\lambda y \lambda z[(y\ z) \wedge$ ((**talk** (**about** z)) **j**)]: CN\CN	= 10

Figure 1: derivation of 'which John talked about'

Example (30a) is derived as shown in Figure 1 in Fitch-style natural deduction **L** with n-ary adjunction.

However, such a treatment does not account for instances such as (31) in which the gap is medial.

(31) (the dog) which$_i$ John saw e_i today

This motivates extension providing for a certain amount of structural flexibility. Structural modalities (referred to as exponentials in linear logic) provide for local encoding of structural facilitation; see Girard (1987); Morrill, Leslie, Hepple and Barry (1990); Barry, Hepple, Leslie and Morrill (1991); Došen (1990); Moortgat and Morrill (1991).

The general strategy is to provide (**S4**) modalities and to restrict application of structural rules to those instances in which the subconfigurations undergoing manipulation contain formulas bearing the requisite modalities. The **S4** rules are that if a configuration including A yields B then so does that configuration with $\Box A$ instead of A, and that a configuration yields $\Box A$ if it yields A and all of its formulas are modal, i.e. bear $\Box$ as the principal operator:

(32) $\dfrac{\Gamma[A] \Rightarrow B}{\Gamma[\Box A] \Rightarrow B}\Box L \qquad \dfrac{\Box\Gamma \Rightarrow A}{\Box\Gamma \Rightarrow A}\Box L$

Modal inference in Fitch-style invokes the notion of modal subderivations (Fitch 1952). We indicate these by double vertical lines. These contain no hypotheses, but there is an import rule the application of which brings a formula into an existing modal subderivation, or creates a new one, and an export rule which carries a formula out of a modal subderivation. Ordinary formulas cannot be iterated into modal subderivations, but otherwise rules apply as usual. For the minimal normal modal logic **K** import must be accompanied by removal of a principal modal connective, and when formulas are exported a $\Box$ is added. For **S4** we say that the modal operator may be optionally removed during import, and is optionally added during export.

(33) $\Box A$
$\quad \| \; (\Box)A \qquad \Box$ imp

(34) $\quad \| \; A$
$(\Box)A \qquad \Box$ exp

Intuitively import corresponds to transport to an arbitrary accessible possible world (and the optionality of **S4** box removal transitivity of accessibility), and export corresponds to a modal affirmation made on the basis of a demonstration in an arbitrary possible world (and the optionality of **S4** box addition reflexivity of accessibility). We shall see treatment of the structural aspects in the examples of the next section.

5.2 Model Theory for Structurally Facilitating Operators

In Morrill (1992a) we introduce models that have structurally distinguished prosodic subalgebras such that conditions like associativity and commutativity hold when one of the participating elements belongs to the relevant subalgebra. We shall discuss such interpretation here, and yet present truly modal proof rules which are incomplete for the subalgebra interpretation.

The remaining two sides of this square are covered by Venema (1993), which provides a complete proof theory for the subalgebra interpretation of structural operators, and by Kurtonina (1993), which provides a modal interpretation of structural operators.

Example: Associativity in NL

For controlled associativity in **NL** we interpret in an algebra $\langle L, +, L' \rangle$ where $\langle L', + \rangle$ is a subalgebra of $\langle L, + \rangle$ such that (35) obtains.

(35) $s_1{+}(s_2{+}s_3) = (s_1{+}s_2){+}s_3$ if s_1, s_2 or $s_3 \in L'$

We refer to this as an association subalgebra. The language $\mathcal{F}$ of category formulas of **NL**+$\{\bigcirc\}$ is freely generated as follows:

(36) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \bigcirc\mathcal{F}$

The multiplicatives are interpreted in $\langle L, + \rangle$ as usual. In addition we have

(37) $D(\bigcirc A) = D(A) \cap L'$

(The semantic dimension of interpretation is inactive with respect to structural modalities, for each of which the type map is $T(\,\Box A) = T(A)$, so there seems little point in spelling this dimension out explicitly.)

In addition to the left and right Gentzen modality rules for $\bigcirc$, there is the operational rule:

(38) $$\frac{\Gamma[((\Delta_1, \Delta_2), \Delta_3)] \Rightarrow A}{\Gamma[(\Delta_1, (\Delta_2, \Delta_3))] \Rightarrow A} \bigcirc\text{A, provided some } \Delta_i \text{ is } \bigcirc\text{-ed}$$

The proof rule condition requires (at least) one of the participating subconfigurations to have all its formulas bearing $\bigcirc$. This corresponds to interpretation in an *association* subalgebra; by an *associative* subalgebra we would mean one for which the association law holds provided all participating elements belong to the subalgebra, and correspondingly all of Δ_i would have to be $\bigcirc$-ed. But it is the former possibility that we shall want to use linguistically.

The Fitch-style rules are thus:

(39) α: $\bigcirc A$
$\quad\|$ α: $(\bigcirc)A$ $\quad\bigcirc$imp

1.	*which* $-$ $\lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)]$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\bigcirc\mathrm{N})$	
2.	*John* $-$ $\mathbf{j}$: N	
3.	*likes* $-$ **like**: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
4.	$a - x$: $\bigcirc$N	H
5.	$\|\ a - x$: N	4 $\bigcirc$imp
6.	$a - x$: N	5 $\bigcirc$exp
7.	$(likes{+}a) - (\mathbf{like}\ x)$: $\mathrm{N}\backslash\mathrm{S}$	3, 6, E/
8.	$(John{+}(likes{+}a)) - ((\mathbf{like}\ x)\ \mathbf{j})$: S	2, 7 E$\backslash$
9.	$((John{+}likes){+}a) - ((\mathbf{like}\ x)\ \mathbf{j})$: S	4, 8 $\bigcirc$A
10.	$(John{+}likes) - \lambda x((\mathbf{like}\ x)\ \mathbf{j})$: S/$\bigcirc$N	4, 9 I/
11.	$(which{+}(John{+}likes)) -$	
	$(\lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)]\ \lambda x((\mathbf{like}\ x)\ \mathbf{j}))$: $\mathrm{CN}\backslash\mathrm{CN}$	1, 10 E/
12.	$(which{+}(John{+}likes)) -$	$=11$
	$\lambda y\lambda z[(y\ z)\wedge((\mathbf{like}\ z)\ \mathbf{j})]$: $\mathrm{CN}\backslash\mathrm{CN}$	

Figure 2: Derivation of 'which John likes' in $\mathbf{NL}{+}\{\bigcirc\}$

(40)
$$\begin{array}{l} \|\ \alpha: A \\ \alpha: (\bigcirc)A \quad \bigcirc\,\mathrm{exp} \end{array}$$

(41)
$$\begin{array}{ll} n. & \beta: \bigcirc B \\ m. & \dfrac{\alpha[((\alpha_1+\alpha_2)+\alpha_3)]: A}{\alpha[(\alpha_1+(\alpha_2+\alpha_3))]: A}\ \bigcirc\mathrm{A}\ n,m\ \beta=\alpha_i \end{array}$$

$$\begin{array}{ll} n. & \beta: \bigcirc B \\ m. & \dfrac{\alpha[(\alpha_1+(\alpha_2+\alpha_3))]: A}{\alpha[((\alpha_1+\alpha_2)+\alpha_3)]: A}\ \bigcirc\mathrm{A}\ n,m\ \beta=\alpha_i \end{array}$$

A minimal example is provided in labelled Fitch-style as shown in Figure 2. We see here why it is the disjunctive as opposed to conjunctive formulation of structural modality which is useful: the latter would require modalities on all the elements required to participate in a restructuring.

This is not enough however to generate medial extraction: S/$\bigcirc$N means an element which combines with an $\bigcirc$N at its right periphery to form an S.

Thus e.g. (42), where the object is missing from before the adverb, would not be generated.

(42) the dog which John saw today

For this commutation would also be required.

5.2.1 Example: Association and commutation in NL

We interpret in an algebra $\langle L, +, L'\rangle$ where $\langle L', +\rangle$ is a subalgebra of $\langle L, +\rangle$ such that

(43) $s_1{+}(s_2{+}s_3) = (s_1{+}s_2){+}s_3$ if s_1, s_2 or $s_3 \in L'$
$s_1{+}s_2 = s_2{+}s_1$ if s_1 or $s_2 \in L'$

The language $\mathcal{F}$ of category formulas of **NL**+$\{\triangle\}$ is (44).

(44) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \triangle\mathcal{F}$

The multiplicatives are interpreted in $\langle L, +\rangle$ as usual. In addition we have

(45) $D(\triangle A) = D(A) \cap L'$

Operational Gentzen rules are as follows:

(46) $$\frac{\Gamma[((\Delta_1, \Delta_2), \Delta_3)] \Rightarrow A}{\Gamma[(\Delta_1, (\Delta_2, \Delta_3))] \Rightarrow A}\triangle\mathrm{A}\text{, provided some } \Delta_i \text{ is } \triangle\text{-ed}$$

(47) $$\frac{\Gamma[(\Delta_1, \Delta_2)] \Rightarrow C}{\Gamma[(\Delta_2, \Delta_1)] \Rightarrow C}\triangle\mathrm{P}\text{, provided some } \Delta_i \text{ is } \triangle\text{-ed}$$

Apart from the rules of import and export, the Fitch-style presentation has:

(48)
$$\begin{array}{ll} n. & \beta\colon \triangle B \\ m. & \alpha[((\alpha_1{+}\alpha_2){+}\alpha_3)]\colon A \\ \hline & \alpha[(\alpha_1{+}(\alpha_2{+}\alpha_3))]\colon A \quad \triangle\mathrm{A}\ n, m\ \beta = \alpha_i \end{array}$$

$$\begin{array}{ll} n. & \beta\colon \triangle B \\ m. & \alpha[(\alpha_1{+}(\alpha_2{+}\alpha_3))]\colon A \\ \hline & \alpha[((\alpha_1{+}\alpha_2){+}\alpha_3)]\colon A \quad \triangle\mathrm{A}\ n, m\ \beta = \alpha_i \end{array}$$

(49) $\begin{array}{ll} n. & \beta\colon \triangle B \\ m. & \alpha[(\alpha_1{+}\alpha_2)]\colon A \\ \cline{2-2} & \alpha[(\alpha_2{+}\alpha_1)]\colon A \end{array}$ $\triangle$P n, m $\beta = \alpha_i$

A derivation is shown in Figure 3.

Example: Commutation in L

We interpret in an algebra $\langle L, +, L' \rangle$ where $\langle L, + \rangle$ is associative and has subalgebra $\langle L', + \rangle$ such that

(50) $s_1{+}s_2 = s_2{+}s_1$ if s_1 or $s_2 \in L'$

The language $\mathcal{F}$ of category formulas of $\mathbf{L}{+}\{\triangle\}$ is given by (51).

(51) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \triangle\mathcal{F}$

The multiplicatives are interpreted in $\langle L, + \rangle$ as usual. In addition we have (52).

(52) $D(\triangle A) = D(A) \cap L'$

Assuming the Gentzen formulation with implicit associativity, the operational rule is:

(53) $$\frac{\Gamma_1, A, B, \Gamma_2 \Rightarrow C}{\Gamma_1, B, A, \Gamma_2 \Rightarrow C}\triangle\text{P, provided } A \text{ or } B \text{ is } \triangle\text{-ed}$$

In Fitch-style the operational rule is (54).

(54) $\begin{array}{ll} n. & \beta\colon \triangle B \\ m. & \alpha[\alpha_1{+}\alpha_2]\colon A \\ \cline{2-2} & \alpha[\alpha_2{+}\alpha_1]\colon A \end{array}$ $\triangle$P n, m $\beta = \alpha_i$

By way of example see Figure 4.

1.	$\mathit{which} - \lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\triangle\mathrm{N})$	
2.	$\mathit{John} - \mathbf{j}$: N	
3.	$\mathit{saw} - \mathbf{see}$: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
4.	$\mathit{today} - \mathbf{today}$: $(\mathrm{N}\backslash\mathrm{S})\backslash(\mathrm{N}\backslash\mathrm{S})$	
5.	$a - x$: $\triangle\mathrm{N}$	H
6.	$a - x$: N	5 $\triangle$ imp
7.	$a - x$: N	6 $\triangle$ exp
8.	$(\mathit{saw}+a) - (\mathbf{see}\ x)$: $\mathrm{N}\backslash\mathrm{S}$	3, 7 E/
9.	$((\mathit{saw}+a)+\mathit{today}) - (\mathbf{today}\ (\mathbf{see}\ x))$: $\mathrm{N}\backslash\mathrm{S}$	4, 8 E\
10.	$(\mathit{John}+((\mathit{saw}+a)+\mathit{today})) -$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	2, 9 E\
11.	$(\mathit{John}+(\mathit{saw}+(a+\mathit{today}))) -$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	5, 10 $\triangle$A
12.	$(\mathit{John}+(\mathit{saw}+(\mathit{today}+a))) -$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	5, 11 $\triangle$P
13.	$(\mathit{John}+((\mathit{saw}+\mathit{today})+a)) -$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	5, 12 $\triangle$A
14.	$((\mathit{John}+(\mathit{saw}+\mathit{today}))+a) -$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	5, 13 $\triangle$A
15.	$(\mathit{John}+(\mathit{saw}+\mathit{today})) -$ $\lambda x((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: $\mathrm{S}/\triangle\mathrm{N}$	5, 14 I/
16.	$(\mathit{which}+(\mathit{John}+(\mathit{saw}+\mathit{today}))) -$ $(\lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]\ \lambda x((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j}))$: $\mathrm{CN}\backslash\mathrm{CN}$	1, 15 E/
17.	$(\mathit{which}+(\mathit{John}+(\mathit{saw}+\mathit{today}))) -$ $\lambda y \lambda z[(y\ z) \wedge ((\mathbf{today}\ (\mathbf{see}\ z))\ \mathbf{j})]$: $\mathrm{CN}\backslash\mathrm{CN}$	= 16

Figure 3: Derivation of ‘which John saw today’ in **NL**+$\{\triangle\}$

1.	*which* $-$ $\lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\triangle\mathrm{N})$	
2.	*John* $-$ $\mathbf{j}$: N	
3.	*saw* $-$ $\mathbf{see}$: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
4.	*today* $-$ $\mathbf{today}$: $(\mathrm{N}\backslash\mathrm{S})\backslash(\mathrm{N}\backslash\mathrm{S})$	
5.	$a - x$: $\triangle\mathrm{N}$	H
6.	$a - x$: N	5 $\triangle$ imp
7.	$a - x$: N	6 $\triangle$ exp
8.	*saw+a* $-$ $(\mathbf{see}\ x)$: $\mathrm{N}\backslash\mathrm{S}$	3, 7 E/
9.	*saw+a+today* $-$ $(\mathbf{today}\ (\mathbf{see}\ x))$: $\mathrm{N}\backslash\mathrm{S}$	4, 8 E\
10.	*John+saw+a+today* $-$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	2, 9 E\
11.	*John+saw+today+a* $-$ $((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: S	5, 10 P$\triangle$
12.	*John+saw+today* $-$ $\lambda x((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j})$: $\mathrm{S}/\triangle\mathrm{N}$	5, 11 I/
13.	*which+John+saw+today* $-$ $(\lambda x \lambda y \lambda z[(y\ z) \wedge (x\ z)]\ \lambda x((\mathbf{today}\ (\mathbf{see}\ x))\ \mathbf{j}))$: $\mathrm{CN}\backslash\mathrm{CN}$	1, 12 E/
14.	*which+John+saw+today* $-$ $\lambda y \lambda z[(y\ z) \wedge ((\mathbf{today}\ (\mathbf{see}\ z))\ \mathbf{j})]$: $\mathrm{CN}\backslash\mathrm{CN}$	= 13

Figure 4: Derivation of 'which John saw today' in $\mathbf{L}+\{\triangle\}$

We shall note how, in addition to facilitating medial extraction, such apparatus enables capture of the curious facts concerning verbs such as 'assure', which exhibit a valency that can be satisfied by extraction, but not by a canonical lexically realised complement (cf. Kayne 1984):

(55) a. (the man) who I assure you to be reliable
b. *I assure you the man to be reliable.

Assignment of the verb to $((\mathrm{N}\backslash\mathrm{S})/\mathrm{VP})/\triangle\mathrm{N}$ characterises the facts.[2] Here it is the modal and not just structural properties of the operator that we are able to exploit. Because $\mathrm{A} \Rightarrow \Box\mathrm{A}$ is not generally valid, lexical material

[2] In Morrill (1992) it is proposed that $(\mathrm{N}\backslash(\mathrm{S}\uparrow\mathrm{N}))/\mathrm{VP}$ suffices when a relative pronoun is $\mathrm{R}/(\mathrm{S}\uparrow\mathrm{N})$. Sag (p.c.) has rightly indicated that that treatment fails when an 'assure'-type verb is subordinate in a relative clause; the version in the text here does not suffer the same problem.

cannot satisfy the modal valency. But a relative pronoun "gap subtype" of course can, bearing as it does the modality.

5.3 Embedding

Girard (1987) shows how a particular translation faithfully embeds intuitionistic logic into linear logic with a (contraction and weakening) modality. Morrill (1992) conjectured that the same translation works generally for the subalgebra interpretations; this is proved in Venema (1993).

5.4 Limitation: Island Constraints

We turn now to the respect in which associative Lambek calculus is linguistically too strong. As it stands it cannot respect constraints such as the fact that coordinate stuctures are islands to extraction (Coordinate Structure Constraint) and that so also are complex noun phrases (e.g. ones with relative clauses; Complex Noun Phrase Constraint), that subjects are islands (Subject Condition),[3] and that the "subject" of a possessive clitic is an island (for such constraints see Ross 1967).:

(56) a. *the man that$_i$ [Mary likes Fred and John dislikes e_i]
b. *the man that$_i$ John met a woman [that$_j$ $e_{j/i}$ loves $e_{i/j}$]
c. ?a topic which$_i$ the professor of likes Mary
d. *the man who$_i$ John read [the brother of e_i's] book

For capture of such constraints we invoke operators not for structural facilitation, but for inhibition.

5.5 Structural Inhibition

Morrill (1992b) proposes introduction of bracketing and antibracketing operators. The category formulas of $\mathbf{L} + \{[\,], [\,]^{-1}\}$ are defined as shown in (57).

[3] In Morrill (1992) the view was addressed that only sentential subjects are islands: a more demanding interpretation of the data since it must then be explained how while subordinate subjects cannot themselves extract from after a complementiser, their subexpressions can. That treatment invokes an existential modality. The treatment in the text here address the more conservative (and more easily characterisable) interpretation of the data.

(57) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid [\,]\mathcal{F} \mid [\,]^{-1}\mathcal{F}$

For interpretation we use algebras $\langle L, +, [\cdot]\rangle$ where $\langle L, +\rangle$ is as for **L** and $[\cdot]$ is a unary operation. Antibracketing is to be defined by reference to the inverse of $[\cdot]$, and to avoid an asymmetric proof theory caused by $[\cdot]^{-1}$ being only a partial operation on L, it is best to require that $[\cdot]$ is 1–1. Then we have:

(58) $[[s]^{-1}] = [[s]]^{-1} = s$

I.e. $[\cdot]$ and $[\cdot]^{-1}$ are a permutation and its inverse. Category formulas are interpreted as usual, together with:

(59)
$$\begin{array}{lcl} D([\,]A) & = & \{[s] | s \in D(A)\} \\ D([\,]^{-1}A) & = & \{s | [s] \in D(A)\} \\ & = & \{[s]^{-1} | s \in D(A)\} \end{array}$$

(Note that the antibracket interpretation can be given in the second manner provided $[\cdot]^{-1}$ is a total function.) Again, the semantic dimension is invariant and we shall omit it except in examples.

The Gentzen sequent configurations $\mathcal{O}$ for $\mathbf{L} + \{[\,], [\,]^{-1}\}$ are defined by mutual recursion with "atomic" configurations $\mathcal{G}$ as follows.

(60)
$$\begin{array}{lcl} \mathcal{O} & = & \mathcal{G}, \ldots, \mathcal{G} \\ \mathcal{G} & = & \mathcal{F} \mid [\mathcal{O}] \mid [\mathcal{O}]^{-1} \end{array}$$

There are the structural rules (61) for sequent metalanguage bracketing and antibracketing (in this context incomplete square brackets are used to indicate distinguished subconfigurations).

(61)
$$\frac{\Gamma\lfloor[[\Delta]^{-1}]\rfloor \Rightarrow A}{\Gamma\lfloor\Delta\rfloor \Rightarrow A}[[\,]^{-1}] \qquad \frac{\Gamma\lfloor[[\Delta]]^{-1}\rfloor \Rightarrow A}{\Gamma\lfloor\Delta\rfloor \Rightarrow A}[[\,]]^{-1}$$

The logical rules are as follows:

(62)
$$\frac{[\Gamma]^{-1} \Rightarrow A}{\Gamma \Rightarrow [\,]A}[\,]R \qquad \frac{\Gamma\lfloor[A]\rfloor \Rightarrow B}{\Gamma\lfloor[\,]A\rfloor \Rightarrow B}[\,]L$$

(63) $$\frac{[\,\Gamma\,] \Rightarrow A}{\Gamma \Rightarrow [\,]^{-1}A}[\,]^{-1}R \qquad \frac{\Gamma\lfloor [\,A\,]^{-1}\rfloor \Rightarrow B}{\Gamma\lfloor [\,]^{-1}A\rfloor \Rightarrow B}[\,]^{-1}L$$

5.5.1 Fitch-style proof theory

We generate labels as follows:

(64) $\mathcal{P} = \mathcal{U} \mid \mathcal{K} \mid \mathcal{P}{+}\mathcal{P} \mid [\,\mathcal{P}\,] \mid [\,\mathcal{P}\,]^{-1}$

There is the following label equation:

(65) $[\,[\,\alpha\,]\,]^{-1} = [\,[\,\alpha\,]^{-1}\,] = \alpha$

And there are the logical rules:

(66) a. $n.\quad \dfrac{\alpha\colon A}{[\,\alpha\,]\colon [\,]A}\ \mathrm{I}[\,]\ n$

b. $n.\quad \dfrac{\alpha\colon A}{[\,\alpha\,]^{-1}\colon [\,]^{-1}A}\ \mathrm{I}[\,]^{-1}\ n$

c. $n.\quad \dfrac{\beta\colon [\,]A}{[\,\beta\,]^{-1}\colon A}\ \mathrm{E}[\,]\ n$

d. $n.\quad \dfrac{\beta\colon [\,]^{-1}A}{[\,\beta\,]\colon A}\ \mathrm{E}[\,]^{-1}\ n$

5.5.2 Embedding

Hypothesis. This allows embedding of **NL** in $\mathbf{L}{+}\{[\,], [\,]^{-1}\}$ (Morrill 1992b): $A \vdash_{\mathbf{NL}} B$ iff $|A| \vdash_{\mathbf{L}+\{[\,],[\,]^{-1}\}} |B|$ where

(67)
$$\begin{array}{lcl} |A\backslash B| & = & |A|\backslash[\,]^{-1}|B| \\ |B/A| & = & [\,]^{-1}|B|/|A| \\ |A\bullet B| & = & [\,](|A|\bullet|B|) \\ |A| & = & A \text{ for atomic } A \end{array}$$

1.	a: $[\,](([\,]^{-1}B/A)\bullet A)$	
2.	$[a]^{-1}$: $([\,]^{-1}B/A)\bullet A$	E[ ] 1
3.	b: $[\,]^{-1}B/A$	H
4.	c: A	H
5.	$b{+}c$: $[\,]^{-1}B$	E/ 3, 4
6.	$[b{+}c]$: B	E$[\,]^{-1}$ 5
7.	$[[a]^{-1}]$: B	E$\bullet$ 2, 3, 4, 6
8.	a: B	= 7
9.	d: $B\backslash[\,]^{-1}C$	H
10.	$a{+}d$: $[\,]^{-1}C$	E\ 8, 9
11.	$[a{+}d]$: C	E$[\,]^{-1}$ 10
12.	$[[a{+}d]]^{-1}$: $[\,]^{-1}C$	I$[\,]^{-1}$ 11
13.	$a{+}d$: $[\,]^{-1}C$	= 12
14.	a: $[\,]^{-1}C/(B\backslash[\,]^{-1}C)$	I/ 9, 13

Figure 5: Embedded proof

For example,

(68) $\mathrm{(B/A)\bullet A} \vdash_{\mathbf{NL}} \mathrm{C/(B\backslash C)}$

$|\mathrm{(B/A)\bullet A}| = [\,](([\,]^{-1}\mathrm{B/A})\ \bullet\mathrm{A})$ and $|\mathrm{C/(B\backslash C)}| = [\,]^{-1}\mathrm{C/(B\backslash}[\,]^{-1}\mathrm{C)}$. The proof is given in Figure 5.

5.5.3 Linguistic Application: Prosodic Islands

In combine structural facilitation and bracketing structural inhibition in $\mathbf{L}+\{[\,], [\,]^{-1}, \triangle\}$ with category formulas given as in (69).

(69) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid [\,]\mathcal{F} \mid [\,]^{-1}\mathcal{F} \mid \triangle\mathcal{F}$

For interpretation we use an algebra $\langle L, +, [\,.\,], L'\rangle$ with $\langle L', +\rangle$ a commutation subalgebra of $\langle L, +\rangle$, and apart from multiplicatives as usual there is:

(70)
$$\begin{array}{lcl} D(\triangle A) & = & D(A) \cap L' \\ D([\,]A) & = & \{[s] \mid s \in D(A)\} \\ D([\,]^{-1}A) & = & \{s \mid [s] \in D(A)\} \end{array}$$

1.	*the*: N/CN	
2.	*professor*: CN	
3.	*of*: (CN\CN)/N	
4.	*physics*: N	
5.	*likes*: ([ ]N\S)/N	
6.	*Mary*: N	
7.	*likes+Mary*: [ ]N\S	6, 5 E/
8.	*of+physics*: CN\CN	4, 3, E/
9.	*professor+of+physics*: CN	2, 8, E/
10.	*the+professor+of+physics* : N	1, 9, E/
11.	[*the+professor+of+physics*] : [ ]N	10 I[ ]
12.	[*the+professor+of+physics*]+*likes+Mary*: S	7, 11, E\

Figure 6: Derivation of 'the professor of physics likes Mary'

Then islands are defined thus:

(71)
and	:=	$(S\backslash[\,]^{-1}S)/S$
likes	:=	$([\,]N\backslash S)/N$
's	:=	$N\backslash[\,]^{-1}(N/CN)$
that	:=	$[\,]^{-1}R/(S/\triangle N)$

There project prosodic forms (72).

(72)
a. [[*Mary*]+*likes+Fred+and*+[*John*]+*dislikes+Bill*]
b. [*John*]+*met+the+woman*+[*that*+[*Bill*]+*loves*]
c. [*The+professor+of+physics*]+*likes+Mary*
d. [*John*]+*read*+[*the+brother+of+Mary+'s*]+*book*

For example, (72c) is derived as shown in Figure 6. Example (72a) is derived as shown in Figure 7.

That the bracketing creates islandood is illustrated by the fact that the inference to line 17 in Figure 8 is not licensed, because the bracketing blocks the concluding line of the preceding subderivation from having the hypothetical prosodic variable as the right-hand operand of a + main connector; the (irrelevant) subject bracketing is suppressed here.

1.	*Mary*: N	
2.	*likes*: $([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
3.	*Fred*: N	
4.	*and*: $(\mathrm{S}\backslash[\,]^{-1}\mathrm{S})/\mathrm{S}$	
5.	*John*: N	
6.	*dislikes*: $([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
7.	*Bill*: N	
8.	*dislikes+Bill*: $[\,]\mathrm{N}\backslash\mathrm{S}$	6, 7 E/
9.	[*John*]: $[\,]\mathrm{N}$	5 I[ ]
10.	[*John*]+*dislikes+Bill*: S	9, 8 E\
11.	*and*+[*John*]+*dislikes+Bill*: $\mathrm{S}\backslash[\,]^{-1}\mathrm{S}$	4, 10 E/
12.	*likes+Fred*: $[\,]\mathrm{N}\backslash\mathrm{S}$	2, 3 E/
13.	[*Mary*]: $[\,]\mathrm{N}$	1 I[ ]
14.	[*Mary*]+*likes+Fred*: S	12, 13 E\
15.	[*Mary*]+*likes+Fred+and*+[*John*]+*dislikes+Bill*: $[\,]^{-1}\mathrm{S}$	11, 14 E\
16.	[[*Mary*]+*likes+Fred+and*+[*John*]+*dislikes+Bill*]: S	13 E$[\,]^{-1}$

Figure 7: Derivation of ‘Mary likes Fred and John dislikes Bill’

6 Bar Operators

An inelegance of the bracket operators is that their interpretation populates the prosodic algebra with all kinds of antibracketing and stacked bracketing which is not exploited linguistically. This suggests searching for a way to depolulate the algebra. The way in which we propose to do that here will also mean that only one structurally inhibiting operator will be required, rather than two.

Both bracketing and antibracketing were necessary because we needed to project islands both from the values of functors, and on to the arguments of functors. The operators were interpreted with respect to an operation and its inverse respectively. The refinement we suggest is that we interpret instead a *single* (prefix) unary operator “bar”, $-$, with respect to an operation $\bar{\ }$ which is *self-inverse*, i.e. which satisfies the law of involution: $\overline{\overline{s}} = s$. The terminology[4] is intended to suggest blocking, barriers, barring, and so forth,

[4] We might, of course, also mention Bar-Hillel!

1.	*which*: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\triangle\mathrm{N})$	
2.	*Mary*: N	
3.	*likes*: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
4.	*Fred*: N	
5.	*and*: $(\mathrm{S}\backslash[\ ]^{-1}\mathrm{S})/\mathrm{S}$	
6.	*John*: N	
7.	*dislikes*: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
8.	a: $\triangle\mathrm{N}$	H
9.	a: N	8 E$\triangle$
10.	$dislikes{+}a$: $\mathrm{N}\backslash\mathrm{S}$	7, 9 E/
11.	$John{+}dislikes{+}a$: S	6, 10 E$\backslash$
12.	$and{+}John{+}dislikes{+}a$: $\mathrm{S}\backslash[\ ]^{-1}\mathrm{S}$	5, 11 E/
13.	$likes{+}Fred$: $\mathrm{N}\backslash\mathrm{S}$	3, 4 E/
14.	$Mary{+}likes{+}Fred$: S	2, 13 E$\backslash$
15.	$Mary{+}likes{+}Fred{+}and{+}John{+}dislikes{+}a$: $[\ ]^{-1}\mathrm{S}$	12, 14 E$\backslash$
16.	$[Mary{+}likes{+}Fred{+}and{+}John{+}dislikes{+}a]$: S	15 E$[\ ]^{-1}$
*17.	$[Mary{+}likes{+}Fred{+}and{+}John{+}dislikes]$: $\mathrm{S}/\triangle\mathrm{N}$	8, 16 I/

Figure 8: Non-derivation of ‘Mary likes Fred and John dislikes’

and to plausibly support involution: if it’s barred that you’re barred, presumably you’re back where you started. There is also a resemblance to bars as in X-bar syntax (Jackendoff 1972) in that both deal with vertical projections and dominance (as opposed to horizontal linear word order), but the latter’s stacking of bar features is precisely what involution collapses: rather than by distinct levels of stacking, discriminations will be made through different kinds of bar (multimodally defined and perhaps with interactions).

The category formulas then are as in (73).

(73) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid -\mathcal{F}$

In the model theory for bar operators we have algebras $\langle L, +, \bar{\cdot}\rangle$ where $\langle L, +\rangle$ is as for **L** and $\bar{\cdot}$ is such that

(74) $\overline{\overline{s}} = s$

I.e. it is self-inverse. Then:

(75) $D(-A) \quad = \quad \{\overline{s} | s \in D(A)\}$

The configurations $\mathcal{O}$ of Gentzen sequents are again defined by mutual recursion with atomic configurations $\mathcal{G}$:

(76)
$$\begin{array}{lcl} \mathcal{O} & = & \mathcal{G}, \ldots, \mathcal{G} \\ \mathcal{G} & = & \mathcal{F} \mid \overline{\mathcal{O}} \end{array}$$

There is the structural rule (77) for sequent metalanguage barring.

(77)
$$\frac{\Gamma\lfloor\overline{\overline{\Delta}}\rfloor \Rightarrow A}{\Gamma\lfloor\Delta\rfloor \Rightarrow A} =$$

The logical rules are as follows:

(78)
$$\frac{\overline{\Gamma} \Rightarrow A}{\Gamma \Rightarrow -A} -R \qquad\qquad \frac{\Gamma\lfloor\overline{A}\rfloor \Rightarrow B}{\Gamma\lfloor -A\rfloor \Rightarrow B} -L$$

6.1 Fitch-style proof theory

Labels are generated as follows:

(79) $\mathcal{P} = \mathcal{U} \mid \mathcal{K} \mid \mathcal{P}{+}\mathcal{P} \mid \overline{\mathcal{P}}$

There is the prosodic label equation (80).

(80) $\overline{\overline{\alpha}} = \alpha$

The logical Fitch rules are:

(81)
$$n.\ \ \frac{\alpha\colon A}{\overline{\alpha}\colon -A}\ \mathrm{I}-\ n$$

$$n.\ \ \frac{\beta\colon -A}{\overline{\beta}\colon A}\ \mathrm{E}-\ n$$

1.	a: $-((-B/A)\bullet A)$	
2.	$\overline{a}$: $(-B/A)\bullet A$	E$-$ 1
3.	$\quad$ b: $-B/A$	H
4.	$\quad$ c: A	H
5.	$\quad$ $b{+}c$: $-B$	E/ $3, 4$
6.	$\quad$ $\overline{b{+}c}$: B	E$-$ 5
7.	$\overline{\overline{a}}$: B	E$\bullet$ $2, 3, 4, 6$
8.	a: B	$= 7$
9.	$\quad$ d: $B\backslash{-C}$	H
10.	$\quad$ $a{+}d$: $-C$	E$\backslash$ $8, 9$
11.	$\quad$ $\overline{a{+}d}$: C	E$-$ 10
12.	$\quad$ $\overline{\overline{a{+}d}}$: $-C$	I$-$ 11
13.	$\quad$ $a{+}d$: $-C$	$= 12$
14.	a: $-C/(B\backslash{-C})$	I/ $9, 13$

Figure 9: Embedded proof

6.1.1 Embedding

Hypothesis. This allows embedding of **NL** in **L**+{–}: $A \vdash_{\mathbf{NL}} B$ iff $|A| \vdash_{\mathbf{L}+\{-\}} |B|$ where

(82)
$$\begin{array}{lcl} |A\backslash B| & = & |A|\backslash{-}|B| \\ |B/A| & = & -|B|/|A| \\ |A\bullet B| & = & -(|A|\bullet|B|) \\ |A| & = & A \text{ for atomic } A \end{array}$$

For example,

(83) $\mathrm{(B/A)\bullet A} \vdash_{\mathbf{NL}} \mathrm{C/(B\backslash C)}$

$\mathrm{|(B/A)\bullet A| = -((-B/A)\ \bullet A)}$ and $\mathrm{|C/(B\backslash C)| = -C/(B\backslash{-C})}$. The proof is given in Figure 9.

6.1.2 Linguistic Application: Prosodic Islands

The structurally inhibiting bar operator can be used in conjunction with structurally facilitating operators to characterise the same phenomena as those treated by bracketing. We define category formulas as follows:

(84) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid -\mathcal{F} \mid \triangle\mathcal{F}$

Interpretation is in an algebra $\langle L, +, \bar{\cdot}, L' \rangle$ with $\langle L', + \rangle$ a commutation subalgebra of the associative $\langle L, + \rangle$, and such that apart from multiplicatives as usual there is interpretation as in (85):

(85)
$$\begin{array}{lcl} D(\triangle A) & = & D(A) \cap L' \\ D(-A) & = & \{\overline{s} \mid s \in D(A)\} \end{array}$$

Then islands are defined thus:

(86)

and	:=	(S\–S)/S
likes	:=	(–N\S)/N
’s	:=	N\–(N/CN)
that	:=	–R/(S/△N)

These generate the following prosodic forms.

(87)
a. $\overline{\overline{\mathit{Mary}}+\mathit{likes}+\mathit{Fred}+\mathit{and}+\overline{\mathit{John}}+\mathit{dislikes}+\mathit{Bill}}$
b. $\overline{\mathit{John}}+\mathit{met}+\mathit{the}+\mathit{woman}+\overline{\mathit{that}+\overline{\mathit{Bill}}+\mathit{loves}}$
c. $\overline{\mathit{The}+\mathit{professor}+\mathit{of}+\mathit{physics}}+\mathit{likes}+\mathit{Mary}$
d. $\mathit{John}+\mathit{read}+\overline{\mathit{the}+\mathit{brother}+\mathit{of}+\mathit{Mary}+\text{'}\mathit{s}}+\mathit{book}$

Since the domain projection and islandhood is obtained in essentially the same, but a simplified, manner we repeat just (87a) in Figure 10.

6.2 Subject extraction

Subject extraction following a complementiser is ungrammatical (the Fixed Subject Constraint of Bresnan 1972, or ‘that’-trace effect):

(88) *(the man) who(m)$_i$ John believes that e_i walks

But subject extraction is grammatical when it is from a sentence which is not complementised:

(89) (the man) who(m)$_i$ John believes e_i walks

A categorial treatment licensing such extraction which follows naturally from those of GPSG and HPSG assigns to a sentence-embedding verb the type

1.	*Mary*: N	
2.	*likes*: $(-\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
3.	*Fred*: N	
4.	*and*: $(\mathrm{S}\backslash-\mathrm{S})/\mathrm{S}$	
5.	*John*: N	
6.	*dislikes*: $(-\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
7.	*Bill*: N	
8.	$\mathit{dislikes}+\mathit{Bill}$: $-\mathrm{N}\backslash\mathrm{S}$	6, 7 E/
9.	$\overline{\mathit{John}}$: $-$N	5 I$-$
10.	$\overline{\mathit{John}}+\mathit{dislikes}+\mathit{Bill}$: S	9, 8 E$\backslash$
11.	$\mathit{and}+\overline{\mathit{John}}+\mathit{dislikes}+\mathit{Bill}$: $\mathrm{S}\backslash-\mathrm{S}$	4, 9 E/
12.	$\mathit{likes}+\mathit{Fred}$: $-\mathrm{N}\backslash\mathrm{S}$	2, 3 E/
13.	$\overline{\mathit{Mary}}$: $-$N	1 I$-$
14.	$\overline{\mathit{Mary}}+\mathit{likes}+\mathit{Fred}$: S	12, 13 E$\backslash$
15.	$\overline{\mathit{Mary}}+\mathit{likes}+\mathit{Fred}+\mathit{and}+\overline{\mathit{John}}+\mathit{dislikes}+\mathit{Bill}$: $-$S	11, 14 E$\backslash$
16.	$\overline{\overline{\mathit{Mary}}+\mathit{likes}+\mathit{Fred}+\mathit{and}+\overline{\mathit{John}}+\mathit{dislikes}+\mathit{Bill}}$: S	15 E$-$

Figure 10: Derivation of 'Mary likes Fred and John dislikes Bill'

$\mathrm{VP}/(\mathrm{S}\vee(\mathrm{VP}\bullet\triangle\mathrm{N}))$ rather than simply VP/S, where VP is $(-\mathrm{N}\backslash\mathrm{S})$. For use of the disjunction see Morrill (1990, 1992), which explains how the semantics is conditioned to the different syntactic categories.. The approach adopted here resolves a puzzle in a common British English dialect which allows the "object" or "accusative" relative pronoun 'whom' to bind downstairs "subject/nominative" positions, but not upstairs ones:

(90) a. (Bill believes) he/*him walks.
b. (the man) whom Bill believes e_i walks
c. *(the man) whom walks

Assignment of type $\mathrm{R}/(\mathrm{S}/\triangle\mathrm{N})$ (or equivalently: $\mathrm{R}/(\triangle\mathrm{N}\backslash\mathrm{S})$) to 'whom' already predicts binding of object and downstairs nominative positions licensed as above, and non-binding of upstairs nominative positions (marked as islands as above). The relative pronoun 'who', which has the wider distribution also binding upstairs nominative positions, is given the wider distribution, but one still prohibiting downstairs 'that'-trace violaton, by assignment to the type the type $\mathrm{R}/((\overline{\mathrm{N}}\sqcap\triangle\mathrm{N})\backslash\mathrm{S})$. (Again, for the conjunction see Morrill 1992;

in this case the semantics is the same for the options that exist as semantic alternatives.)

6.3 Differential Penetrability of Islands

Domains may be penetrated by some fillers, but not by others:

(91) a. the man who$_i$ Mary went to London [without speaking to e_i]
b. *the man [to whom]$_i$ Mary went to London [without speaking e_i]

To deal with such a situation we invoke two different notions of bracketing and permutation: weak and strong, with the former subscripted by %. Category formulas $\mathcal{F}$ are generated thus:

(92) $\mathcal{F} = \mathcal{A} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid -_{\%}\mathcal{F} \mid -\mathcal{F} \mid \triangle_{\%}\mathcal{F} \mid \triangle\mathcal{F}$

For prosodic interpretation we use an algebra $\langle L, +, \bar{\cdot}^{\%}, \bar{\cdot}, L^{\%}, L'\rangle$ such that $\langle L, +\rangle$ is a semigroup, and $\langle L^{\%}, +\rangle$ and $\langle L', +\rangle$ are commutation subalgebras of $\langle L, +\rangle$ such that (93) obtains.

(93)
$$\overline{s}^{\%}+s' = \overline{s+s'}^{\%} \text{ if } s' \in L'$$
$$s'+\overline{s}^{\%} = \overline{s'+s}^{\%} \text{ if } s' \in L'$$

Apart from the usual interpretation of multiplicatives, we have:

(94)
$$\begin{array}{lcl} D(\triangle_{\%}A) & = & D(A) \cap L^{\%} \\ D(\triangle A) & = & D(A) \cap L' \\ D(-_{\%}A) & = & \{\overline{s}^{\%} \mid s \in D(A)\} \\ D(-A) & = & \{\overline{s} \mid s \in D(A)\} \end{array}$$

Labelled Fitch-style structural rules for the permutors are as follows.

(95) a.
$$\begin{array}{ll} m. & \beta\text{: } \triangle_{\%}B \\ n. & \dfrac{\alpha[\beta_1{+}\beta_2]\text{: } A}{\alpha[\beta_2{+}\beta_1]\text{: } A} \ \triangle_{\%}\mathrm{P}\ m,n,\ \beta = \beta_1 \text{ or } \beta_2 \end{array}$$

b.
$$\begin{array}{ll} m. & \beta\text{: } \triangle B \\ n. & \dfrac{\alpha[\beta_1{+}\beta_2]\text{: } A}{\alpha[\beta_2{+}\beta_1]\text{: } A} \ \triangle\mathrm{P}\ m,n,\ \beta = \beta_1 \text{ or } \beta_2 \end{array}$$

(96) a.
$$\frac{\begin{array}{ll} m. & \beta\colon \triangle B \\ n. & \alpha[\overline{\beta_1}^{\%}{+}\beta_2]\colon A \end{array}}{\alpha[\overline{\beta_1{+}\beta_2}^{\%}]\colon A} \triangle\mathrm{B}\ m, n, \beta = \beta_1 \text{ or } \beta_2$$

b.
$$\frac{\begin{array}{ll} n. & \beta\colon \triangle B \\ m. & \alpha[\overline{\beta_1{+}\beta_2}^{\%}]\colon A \end{array}}{\alpha[\beta_1{+}\overline{\beta_2}^{\%}]\colon A} \triangle\mathrm{B}\ n, m, \beta = \beta_1 \text{ or } \beta_2$$

(97) a.
$$\frac{\begin{array}{ll} m. & \beta\colon \triangle B \\ n. & \alpha[\beta_1^{\%}{+}\overline{\beta_2}]\colon A \end{array}}{\alpha[\overline{\beta_1{+}\beta_2}^{\%}]\colon A} \triangle\mathrm{B}\ m, n, \beta = \beta_1 \text{ or } \beta_2$$

b.
$$\frac{\begin{array}{ll} n. & \beta\colon \triangle B \\ m. & \alpha[\overline{\beta_1{+}\beta_2}^{\%}]\colon A \end{array}}{\alpha[\overline{\beta_1}{+}\beta_2^{\%}]\colon A} \triangle\mathrm{B}\ n, m, \beta = \beta_1 \text{ or } \beta_2$$

Then e.g. given the following ‘whom’ but not ‘to whom’ can bind into the adverbial (and neither can bind into relative clauses, cf. the CNPC).

(98)

to whom	:=	$-\mathrm{R}/(\mathrm{S}/\triangle_{\%}\mathrm{PP})$
who	:=	$-\mathrm{R}/(\mathrm{S}/\triangle\mathrm{N})$
without	:=	$-_{\%}((\mathrm{N}\backslash\mathrm{S})\backslash(\mathrm{N}\backslash\mathrm{S}))/\mathrm{VP}$

A canonical form is generated as shown in Figure 11. The nominal extraction is obtained as in Figure 12. The corresponding prepositional extraction is not obtained because the weak boundary penetrating equation between lines 14 and 15 depends on the prosodic variable a of the subderivation hypothesis at line 7 being of $\triangle$-type, whereas the gap subtype of the prepositional filler is only of $\triangle_{\%}$-type.[5]

[5] In Morrill (1992) the differential penetration is treated by through use of $\uparrow$ for extraction. However that “one step” means of extraction, as opposed to the “localised step” means using structural modalities, seems to have a less natural interaction with structural inhibition.

1.	*Mary*: N	
2.	*went+to+London*: $N\backslash S$	
3.	*without*: $-_{\%}((N\backslash S)\backslash(N\backslash S))/VP$	
4.	*speaking*: VP/PP	
5.	*to*: PP/N	
6.	*John*: N	
7.	*to+John*: PP	E/ 5, 6
8.	*speaking+to+John*: VP	E/ 4, 7
9.	*without+speaking+to+John*: $-_{\%}((N\backslash S)\backslash(N\backslash S))$	E/ 3, 8
10.	$\overline{\textit{without+speaking+to+John}}^{\%}$: $(N\backslash S)\backslash(N\backslash S)$	$E-_{\%}$ 9
11.	*went+to+London+*$\overline{\textit{without+speaking+to+John}}^{\%}$: $N\backslash S$	E\ 2, 10
12.	*Mary+went+to+London+* $\overline{\textit{without+speaking+to+John}}^{\%}$: S	E\ 1, 11

Figure 11: Derivation of ‘Mary went to London without speaking to John’

1.	who: $-\mathrm{R}/(\mathrm{S}/\triangle\mathrm{N})$	
2.	Mary: N	
3.	$\mathit{went{+}to{+}London}$: $\mathrm{N}\backslash\mathrm{S}$	
4.	$\mathit{without}$: $-_{\%}((\mathrm{N}\backslash\mathrm{S})\backslash(\mathrm{N}\backslash\mathrm{S}))/\mathrm{VP}$	
5.	$\mathit{speaking}$: VP/PP	
6.	to: PP/N	
7.	a: $\triangle\mathrm{N}$	H
8.	a: N	E$\triangle$ 7
9.	$\mathit{to{+}a}$: PP	E/ 6, 8
10.	$\mathit{speaking{+}to{+}a}$: VP	E/ 5, 9
11.	$\mathit{without{+}speaking{+}to{+}a}$: $-_{\%}((\mathrm{N}\backslash\mathrm{S})\backslash(\mathrm{N}\backslash\mathrm{S}))$	E/ 4, 10
12.	$\overline{\mathit{without{+}speaking{+}to{+}a}}^{\%}$: $(\mathrm{N}\backslash\mathrm{S})\backslash(\mathrm{N}\backslash\mathrm{S})$	E$-_{\%}$ 11
13.	$\mathit{went{+}to{+}London{+}}$ $\overline{\mathit{without{+}speaking{+}to{+}a}}^{\%}$: $\mathrm{N}\backslash\mathrm{S}$	E$\backslash$ 3, 12
14.	$\mathit{Mary{+}went{+}to{+}London{+}}$ $\overline{\mathit{without{+}speaking{+}to{+}}}^{\%}$: S	E$\backslash$ 2, 13
15.	$\mathit{Mary{+}went{+}to{+}London{+}}$ $\overline{\mathit{without{+}speaking{+}to}}^{\%}\mathit{{+}a}$: S	= 14
16.	$\mathit{Mary{+}went{+}to{+}London{+}}$ $\overline{\mathit{without{+}speaking{+}to}}^{\%}$: $\mathrm{S}/\triangle\mathrm{N}$	I/ 7, 15
17.	$\mathit{who{+}Mary{+}went{+}to{+}London}$ ${+}\overline{\mathit{without{+}speaking{+}to}}^{\%}$: $-^{-1}\mathrm{R}$	E/ 1, 16

Figure 12: Derivation of ‘who Mary went to London without speaking to’

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