Chapter 1

DUTCH WORD ORDER AND BINDING*

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Abstract
This paper offers a type-logical account of Dutch word order with special reference to quantifier binding in subordinate clauses.

We see that the finite tensed verb appears clause-finally. When the main verb is an infinitival complement of a modal or control verb, a so-called verb raising trigger, it appears after the verb raising trigger(s) in a clause-final verb cluster:

a. \((\ldots \text{dat})\) Jan zong  \\
\((\ldots \text{that})\) J. sang  \\
\("(\ldots \text{that})\) Jan sang"

b. \((\ldots \text{dat})\) Jan boeken las  \\
\((\ldots \text{that})\) J. books read  \\
\("(\ldots \text{that})\) Jan read books"

c. \((\ldots \text{dat})\) Jan boeken aan Marie gaf  \\
\((\ldots \text{that})\) J. books to M. gave  \\
\("(\ldots \text{that})\) Jan gave books to Marie"

We see that the finite tensed verb appears clause-finally. When the main verb is an infinitival complement of a modal or control verb, a so-called verb raising trigger, it appears after the verb raising trigger(s) in a clause-final verb cluster:

a. \((\ldots \text{dat})\) Jan boeken kan lezen  \\
\((\ldots \text{that})\) J. books is able read  \\
\("(\ldots \text{that})\) Jan is able to read books"

b. \((\ldots \text{dat})\) Jan boeken wil kunnen lezen  \\
\((\ldots \text{that})\) J. books wants be able read  \\
\("(\ldots \text{that})\) Jan wants to be able to read books"

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The semantic form of (1.2a) is (able (read books)) and that of (1.2b) is (want (able (read books))). Thus we have a syntactic/semantic mismatch with the discontinuous constituent ‘boeken ... lezen’ interpreted as a semantic unit (read books), with an indefinite number of verb raising triggers intervening.

If such a subordinate clause contains a quantifier, there is scope ambiguity as to whether the quantifier takes scope within or outside of the verb raising trigger:

\[(\ldots \text{dat}) \text{ Jan alles wil lezen} \]
\[(\ldots \text{that}) \text{ J. everything wants read} \]
\[\text{“(\ldots that) Jan wants to read everything”} \]

Main clause yes/no interrogative word order, V1 word order, is derived from subordinate clause word order by fronting the finite verb:

\[
\text{Wil Jan boeken lezen.} \\
\text{“Does Jan want to read books?”} \text{ (1.4)}
\]

Main clause declarative word order, V2 word order, is further derived by fronting a major constituent:

\[
\text{Jan wil boeken lezen.} \\
\text{“Jan wants to read books.”} \text{ (1.5)}
\]

In this paper we offer an account of these various facts of Dutch word order.

1. ASSOCIATIVE LAMBEK CALCULUS

The associative Lambek calculus with product unit (Lambek 1988) is the calculus of a monoid \((L, +, \epsilon)\) where + is seen as the associative operation of concatenation: \(s_1 + (s_2 + s_3) = (s_1 + s_2) + s_3\) and \(\epsilon\) is seen as the empty string: \(s + \epsilon = \epsilon + s = s\). The category type formulas \(\mathcal{F}\) are defined on the basis of a set \(\mathcal{A}\) of atomic category type formulas by the type forming operators I (product unit), \(\backslash\) (“under”), \(/\) (“over”) and \(\bullet\) (“product”) as follows:

\[
\mathcal{F} ::= A | I | \mathcal{F}\backslash\mathcal{F} | \mathcal{F}/\mathcal{F} | \mathcal{F}\bullet\mathcal{F} \tag{1.6}
\]

Each category type formula \(A\) is interpreted as a set \(D(A) \subseteq L\) as follows:

\[
D(I) = \{\epsilon\} \\
D(A \backslash B) = \{s | \forall s' \in D(A), s' + s \in D(B)\} \\
D(B / A) = \{s | \forall s' \in D(A), s + s' \in D(B)\} \\
D(A \bullet B) = \{s_1 + s_2 | s_1 \in D(A) & s_2 \in D(B)\} \tag{1.7}
\]
Where we write $\alpha \colon A$ to indicate that expression $\alpha$ is in category $A$ the following deduction rules are valid:

$$
\begin{align*}
\frac{\alpha \colon A}{\alpha + \gamma : B \quad \gamma : A \setminus B}{E} \\
\frac{\gamma : B \setminus A \quad \alpha : A}{\gamma + \alpha : B \quad /E} \\
\frac{a : A}{\gamma + a : B \quad /\!/+} \\
\frac{\alpha + \gamma : B \quad a \colon A}{\alpha : A \quad a \colon A \setminus B \quad \gamma : A \setminus B \quad /\!/+} \\
\end{align*}
$$

Equation (1.8)

$$
\begin{align*}
\frac{\gamma : B \setminus A \quad \alpha : A}{\gamma + \alpha : B \quad /E} \\
\frac{\gamma + a : B \quad a \colon A}{\gamma : B \setminus A \quad /\!/+} \\
\end{align*}
$$

Equation (1.9)

The overline in $\setminus I$ and $/I$ indicates cancellation of a hypothesis with the coindex rule. The rules for $\bullet E$ and $IE$, which are more difficult to formulate, are excluded since they are not needed in this paper.

2. DISCONTINUITY

Versmissen (1991) and Solias (1992, 1994, 1996) treat discontinuity in terms of split strings. Solias works with an algebra $(L, +, \langle \cdot, \cdot \rangle)$ where $+$ is associative and $\langle \cdot, \cdot \rangle$ is “strictly non-associative”, i.e. $(L, +)$ is a semigroup and $(L, \langle \cdot, \cdot \rangle)$ is a free groupoid. Wrap is derived as a parcial operation. This is made more explicit in Morrill and Solias (1993) where the algebra is augmented with projection functions, $(L, +, \langle \cdot, \cdot \rangle, 1, 2)$, such that $1\langle s_1, s_2 \rangle = s_1$ and $2\langle s_1, s_2 \rangle = s_2$. Then wrapping, $W$, is defined as a total operation by $s_1Ws_2 =_{df} 1s_1 + s_2 + 2s_1$, whence infixation, extraction and discontinuous product operators are interpreted by:

$$
\begin{align*}
D(A \uparrow B) &= \{ s_2 \mid \forall s_1 \in D(A), s_1Ws_2 \in D(B) \} \\
D(B \downarrow A) &= \{ s_1 \mid \forall s_2 \in D(B), s_2Ws_1 \in D(A) \} \\
D(A \land B) &= \{ s_1Ws_2 \mid s_1 \in D(A) & s_2 \in D(B) \}
\end{align*}
$$

Equation (1.12)

However, these formulations encounter the technical difficulty of the incompleteness of the non-associative Lambek calculus for free groupoids (Venema 1994), therefore Morrill (1994, 1995) proposes wrap as a primitive operation...
in an algebra \((L, +, (\cdot, \cdot), W)\) where \((L, +)\) is a semigroup and \((L, (\cdot, \cdot))\) and \((L, W)\) are groupoids such that \((s_1, s_2)W s = s_1 + s + s_2\). Nevertheless this does not assure all the expected behaviour of wrap, for example it does not validate the intuitively valid Geach-like shift:

\[
(B \uparrow C) \downarrow D \Rightarrow ((A \setminus B) \uparrow C) \downarrow (A \setminus D)
\] (1.13)

Thus in Morrill (1995, appendix) and Morrill and Merenciano (1996) an alternative tack is taken which consists in sorting the categorial types. The concatenation adjunction has functionality \(L \rightarrow L\). We further define an interpolation adjunction \(W\) of functionality \(L^2 \rightarrow L\): \((s_1, s_2)W s = s_1 + s + s_2\). We refer to sort \(L\) as sort string, and sort \(L^2\) as sort split string. Let us assume that atomic formulas \(A\) are of sort string. The well-sorted category formulas or types \(F\) of sort string and \(F^2\) of sort split string are defined by mutual recursion thus:

\[
\begin{align*}
F & := A \mid I \mid F \setminus F \mid F/F \mid F \star F \mid F^2 \downarrow F \mid F^2 \circ F \\
F^2 & := F \uparrow F
\end{align*}
\] (1.14)

Each formula \(A\) of sort string has an interpretation \(D(A) \subseteq L\) and each formula \(A\) of sort split string has an interpretation \(D(A) \subseteq L^2\):

\[
\begin{align*}
D(A \downarrow B) &= \{s| \forall (s_1, s_2) \in D(A), s_1 + s + s_2 \in D(B)\} \\
D(B \uparrow A) &= \{(s_1, s_2)| \forall s \in D(A), s_1 + s + s_2 \in D(B)\} \\
D(A \sqcap B) &= \{s_1 + s + s_2| (s_1, s_2) \in D(A) \& s \in D(B)\}
\end{align*}
\] (1.15)

Valid deduction rules are as follows:

\[
\begin{align*}
\frac{\vdots \vdots}{\alpha_1 + \alpha_2: B} \quad \frac{\gamma: A \downarrow B}{\alpha_1 + \gamma + \alpha_2: B} \quad \Downarrow E \\
\frac{\vdots \vdots}{\gamma_1 + \alpha + \gamma_2: B} \quad \frac{\alpha: A}{\gamma_1 + \alpha + \gamma_2: B} \quad \Uparrow E \\
\frac{\vdots \vdots}{\alpha_1, \alpha_2: A} \quad \frac{\beta: B}{\alpha_1 + \beta + \alpha_2: A \circ B} \quad \Downarrow I
\end{align*}
\] (1.16)
3. QUANTIFICATION

Quantification is derived by assigning quantifier phrases type \((S \uparrow N) \downarrow S\). Consider the narrow scope (non-specific) and wide scope (specific) readings of ‘John thinks someone walks’. The narrow scope reading is derived by infixation at the level of the subordinate clause as follows:

\[
\begin{align*}
\frac{\alpha: N \quad \text{walks}: N \downarrow S}{\alpha + \text{walks}: S} \quad \exists E \\
\frac{(x, \text{walks}): S \uparrow N \quad \text{someone}: (S \uparrow N) \downarrow S}{\text{thinks}: (N \downarrow S) \downarrow S } \quad \exists E \\
\frac{\text{John}: N \quad \text{thinks} + \text{someone} + \text{walks}: N \downarrow S}{\exists E} \\
\end{align*}
\]
(1.19)

In the wide scope derivation on the other hand, infixation is at the level of the superordinate clause:

\[
\begin{align*}
\frac{\alpha: N \quad \text{walks}: N \downarrow S}{\alpha + \text{walks}: S} \quad \exists E \\
\frac{\text{think}: (N \downarrow S) \downarrow S \quad \exists \exists (x, y)(x y)}{\lambda x \exists y(x y)} \quad \exists E \\
\frac{\exists y(\text{walk } x)}{\lambda x \exists y(x y)} \quad \exists E \\
\frac{\text{John}: N \quad \text{think} \exists y(\text{walk } y) \quad \exists \exists (x, y)(x y)}{\exists E} \\
\end{align*}
\]
(1.20)

Semantics is given by the Curry-Howard rendering of categorial deductions: functional application for rules of implicational use (elimination) and functional abstraction for rules of implicational proof (introduction); node for node with (1.19):

\[
\begin{align*}
\frac{\lambda x (\text{walk } x)}{\exists y(\text{walk } y)} \quad \exists E \\
\frac{\text{think} \exists y(\text{walk } y)}{\lambda x \exists y(x y)} \quad \exists E \\
\frac{\exists y(\text{walk } y)}{\lambda x \exists y(x y)} \quad \exists E \\
\frac{\lambda x \exists y(x y) \quad \exists \exists (x, y)(x y)}{\exists E} \\
\end{align*}
\]
6

Node for node, the semantics is thus:

\[
\begin{align*}
\neg 1 & \quad \text{walk} \\
\text{think} (\text{walk } x) & \quad \neg E \\
(\text{think} (\text{walk } x)) & \quad \neg E \\
\lambda x ((\text{think} (\text{walk } x)) j) & \quad \lambda x \exists y (x y) \\
\exists y ((\text{think} (\text{walk } y)) j) & \quad \neg E \\
\end{align*}
\]

(1.22)

4. FRONTING

Fronting such as relativisation can be treated by assignment of the relative pronoun to \((\text{CN} \setminus \text{CN})/((S \uparrow N) \circ I)\). For example ‘(the book) that Mary sent to John’ is derived thus:

\[
\begin{align*}
\text{that} & : R / ((S \uparrow N) \circ I) \\
\text{Mary} & + \text{sent} + \text{to} + \text{John} : (S \uparrow N) \circ I \\
\text{sent} : ((N \setminus S)/PP)/N \ a : N \\
\text{sent} + a & : (N \setminus S)/PP \quad \text{to} + J : PP \\
\text{M} : N & \quad \text{sent} + a + \text{to} + \text{John} : N \setminus S \\
\end{align*}
\]

(1.23)

Fronting such as topicalisation can be derived by assignment to the empty string of \((X \setminus S)/(( S \uparrow X) \circ I)\). For example ‘The book, Mary sent to John’ is derived thus:

\[
\begin{align*}
\text{the} + \text{book} : N & \quad \text{Mary} + \text{sent} + \text{to} + \text{John} : N \setminus S \\
\end{align*}
\]

(1.24)
5. GEACH EFFECT

The intuitively valid Geach-like shift (1.13) is derivable as follows:

\[
\begin{array}{c}
\frac{(a_1, a_2): (A \setminus B) \uparrow C \quad c: C}{\textbf{1}} \\
\frac{b: A \quad a_1 + c + a_2: A \setminus B}{\textbf{2}} \\
\frac{b + a_1 + c + a_2: B}{\textbf{3}} \\
\frac{(b + a_1, a_2): B \uparrow A \quad \alpha: (B \uparrow C) \downarrow D}{\textbf{4}} \\
\frac{b + a_1 + a_2: D}{\textbf{5}} \\
\frac{\alpha: (A \setminus B) \uparrow C \downarrow (A \setminus D)}{\textbf{6}}
\end{array}
\]

(1.25)

6. DUTCH VERB RAISING PLUS QUANTIFIER RAISING: MULTIPLE DISCONTINUITY

Consider the following example with quantification in a subordinate clause:

(\ldots dat) Jan alles aan Marie wil given
(\ldots that) J. everything to M. wants give

“(\ldots that) Jan wants to give everything to Marie

\[(\ldots dat) \text{Jan alles aan Marie wil given} \quad (1.26)\]

A wide scope derivation is obtained on the following pattern:

\begin{align*}
&\text{a. _ aan Marie _ geven} \\
&\text{b. _ aan Marie wil geven} \\
&\text{c. Jan _ aan Marie wil geven} \\
&\text{d. Jan alles aan Marie wil geven}
\end{align*}

(1.27)

A narrow scope derivation is obtained on the following pattern:

\begin{align*}
&\text{a. _ aan Marie _ geven} \\
&\text{b. _ alles aan Marie _ geven} \\
&\text{c. _ alles aan Marie wil given} \\
&\text{d. Jan alles aan Marie wil given}
\end{align*}

(1.28)

The essential puzzle is the management of multiple points of discontinuity. We treat this by adding to the type language a logical constant \( J \) which is interpreted as a subset \( J \) of the underlying algebraic field such that \( \epsilon \in J \). We refer to this as the connection set.

Intuitively, connections allow for hypothetical reasoning over discontinuous strings as if they were continuous, by supposing that they are connected; the connection set includes \( \epsilon \) because the empty string always connects adjacent
strings. Semantically, J is neutral, like I. It has the following rule of introduction:

\[ e: J1 \]

(1.29)

7. **FINITE AND INFINITE VERBS**

For the generation of simple Dutch subordinate clauses let us assume the following assignments:

\[
\begin{align*}
\text{aan} & \rightarrow m \\
\text{boeken} & \rightarrow \text{books} \\
\text{gaf} & \rightarrow \text{gave} \\
\text{Jan} & \rightarrow j \\
\text{las} & \rightarrow \text{read} \\
\text{zong} & \rightarrow \text{sang}
\end{align*}
\]

(1.30)

These derive in a straightforward way the following:

a. (\ldots dat) Jan zong
   “(\ldots that) Jan sang”

b. (\ldots dat) Jan boeken las
   “(\ldots that) Jan read books” \hspace{1cm} (1.31)

c. (\ldots dat) Jan boeken aan Marie gaf
   “(\ldots that) Jan gave books to Marie”

The possibility of verb-complement discontinuity will be accommodated by having infinite verbs combine with a connection to the left. Where a finite verb has category \( X \), the corresponding infinite verb has category \( J \backslash X \):

\[
\begin{align*}
\text{geven} & \rightarrow \text{give} \\
& \rightarrow J \backslash (PP \backslash (N \backslash (N \backslash S))) \\
\text{lezen} & \rightarrow \text{read} \\
& \rightarrow J \backslash (N \backslash (N \backslash S)) \\
\text{zingen} & \rightarrow \text{sing} \\
& \rightarrow J \backslash (N \backslash S)
\end{align*}
\]

(1.32)
Note that this allows bare infinitival clauses to be generated as continuous strings:

\[
\begin{array}{c}
\text{Jan: } N \\
\text{zingen: } N \setminus (N \setminus S) \vee E \\
\hline
\text{Jan + zingen: } S
\end{array}
\]

(1.33)

\[
\begin{array}{c}
\text{Jan: } N \\
\text{lezen: } N \setminus (N \setminus S) \vee E \\
\hline
\text{Jan + boeken + lezen: } S
\end{array}
\]

(1.34)

8. FINITE AND INFINITE VERB RAISING TRIGGERS

A finite verb raising trigger is defined to infix at the connection site of a verb phrase:

\[
\begin{array}{c}
\text{kan} \quad \text{– is-able} \\
:= ((N \setminus S) \uparrow J) \downarrow (N \setminus S) \\
\hline
\text{wil} \quad \text{– wants} \\
:= ((N \setminus S) \uparrow J) \downarrow (N \setminus S)
\end{array}
\]

(1.35)

This situates the verb raising trigger at the left of an infinitival verb:

\[
\begin{array}{c}
\text{Jan: } N \\
\text{zingen: } J \setminus (N \setminus S) \vee E \\
\hline
\text{kan + zingen: } N \setminus S
\end{array}
\]

\[
\begin{array}{c}
\text{Jan: } N \\
\text{lezen: } N \setminus (N \setminus S) \vee E \\
\hline
\text{boeken + kan + lezen: } N \setminus S
\end{array}
\]

(1.36)

(1.37)
As was the case for ordinary verbs, so it is for verb raising triggers that where the finite verb has category $X$, the infinite verb has category $J \backslash X$:

\[
\begin{align*}
\text{kunnen} & \quad \text{\textit{be-able}} \\
& \quad := J \backslash ((N \backslash S) \uparrow J, (N \backslash S)) \\
\text{wollen} & \quad \text{\textit{want}} \\
& \quad := J \backslash ((N \backslash S) \uparrow J, (N \backslash S))
\end{align*}
\] (1.39)

This accounts for multiple verb raising as in the following, where VP abbreviates $N \backslash S$:

\[
\begin{align*}
(\ldots \text{dat}) \text{ Jan alles \text{ wil lezen}} \\
(\ldots \text{dat}) \text{ J. \text{ everything wants read}} \\
\ldots \text{(\ldots that) \text{ Jan wants to read everything}}
\end{align*}
\] (1.41)

9. VERB RAISING PLUS QUANTIFIER RAISING

Recall the following example:

\[
\begin{align*}
& (\ldots \text{dat}) \text{ Jan alles \text{ wil lezen}} \\
& (\ldots \text{that}) \text{ J. \text{ everything wants read}} \\
& \text{“(\ldots that) \text{ Jan wants to read everything}”}
\end{align*}
\]
The wide scope reading for the quantifier is derived thus, where the quantifier joins the derivation after ‘wil’:

\[ \text{Jan: } N \]
\[ \text{alleen: } N \]
\[ \text{Jan + alleen + wil: } S \]

The narrow scope reading is derived thus, where the quantifier joins the derivation before ‘wil’:

\[ \text{Jan: } N \]
\[ \text{alleen: } N \]
\[ \text{Jan + alleen + wil: } S \]

10. V1 AND V2

For V1 yes/no interrogative order we assume lexical lifting of finite verbs (cf. Hepple 1990):

\[ \alpha - \phi : \equiv f_vA \]
\[ \alpha - \lambda x \phi : \equiv M / (S \uparrow A) \cup J \]
Hence the following, where $X$ is $J \backslash (N \backslash (N \backslash S))$.

\[
\begin{array}{l}
\text{Jan: } N \\
\text{booken: } N \rightarrow \text{lezen: } N \backslash (N \backslash S) \\
\text{will: } M \backslash (\Sigma \times X) \circ J \\
\text{Jan + booken + lezen: } N \backslash T \\
\text{Jan + will + booken + lezen: } M
\end{array}
\]

(1.45)

And for V2 declarative order we further assume assignment to the empty string as follows:

\[
\epsilon \vdash \lambda x. \lambda y (y \ x) := (X \backslash T) / ((M \uparrow X) \circ J)
\]

(1.46)

Thus:

\[
\begin{array}{l}
\text{Jan: } N \\
\text{will + booken + lezen: } N \backslash T \\
\text{Jan + will + booken + lezen: } T
\end{array}
\]

(1.47)

In this way the basic facts of Dutch subordinate and main clause word order are accommodated, including the case of quantifier binding.
Appendix: Sequent calculus for sorted discontinuity

In this appendix we present sequent calculus for sorted discontinuity (cf. Morrill 1998). The basic idea is to represent split strings by two formula occurrences at their two loci of action. These components are punctuated as roots. Sequents come in two sorts. A sort string sequent $\Sigma$ has a sort string succedent on the right and a sort string configuration $O$ on the left. A sort split string sequent $\Sigma^2$ has a sort split string succedent on the right and a sort split string configuration $O^2$ on the left. The well-formed configurations and sequents of sort string and split string are defined as follows:

\[
\begin{align*}
O & ::= [F, O, F, O, F] \mid \sqrt{F^2}, O, \sqrt{F^2} \\
O^2 & ::= \sqrt{F^2} [F, O^2, F, O^2, \sqrt{F^2}, O^2, \sqrt{F^2} \\
\Sigma & ::= O \Rightarrow F \\
\Sigma^2 & ::= O^2 \Rightarrow \sqrt{F^2}
\end{align*}
\] (A.1)

We have sort string and sort split string identity rules. The notation $\Gamma(\Delta)$ indicates a configuration $\Gamma$ with a distinguished subconfiguration $\Delta$. $Y$ and $Z$ vary over formulas of sort string and the roots of formulas of sort split string.

\[
\begin{align*}
\frac{\square A, \sqrt{A}, \sqrt{A} \Rightarrow \sqrt{A}}{\text{id}^2} & \quad \frac{\Gamma(\sqrt{A}) \Rightarrow \sqrt{A}}{\text{id}^1}
\end{align*}
\] (A.2)

The rules for the Lambek connectives are thus:

\[
\begin{align*}
\frac{\text{id}^1}{} & \quad \frac{\text{id}^2}{} \\
\frac{\Gamma(\Delta \Rightarrow Y)}{\Gamma(\Delta A) \Rightarrow Y} & \quad \frac{\Gamma(\Delta(\sqrt{A}, A, \sqrt{A})) \Rightarrow Y}{\Gamma(\Delta(\Delta A)) \Rightarrow Y} \\
\frac{\Delta \Rightarrow Y}{\Delta(\Delta A) \Rightarrow Y} & \quad \frac{\Delta \Rightarrow Y}{\Delta(\Delta A) \Rightarrow Y}
\end{align*}
\] (A.3)

The rules for the discontinuity connectives are as follows:

\[
\begin{align*}
\frac{\sqrt{A}, \Gamma, \sqrt{A} \Rightarrow B}{\text{IR}} & \quad \frac{\Gamma(\sqrt{A}) \Rightarrow \sqrt{A}}{\text{IL}} \\
\frac{\Gamma(\Delta(\sqrt{A}, A, \sqrt{A})) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta A)) \Rightarrow Z}{\Delta \Rightarrow Z} \\
\frac{\Gamma(\Delta A) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta A)) \Rightarrow Z}{\Delta \Rightarrow Z}
\end{align*}
\] (A.4)

The rules for the discontinuity connectives are as follows:

\[
\begin{align*}
\frac{\sqrt{A}, \Gamma, \sqrt{A} \Rightarrow B}{\text{IR}} & \quad \frac{\Gamma(\sqrt{A}) \Rightarrow \sqrt{A}}{\text{IL}} \\
\frac{\Gamma(\Delta(\sqrt{A}, A, \sqrt{A})) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta A)) \Rightarrow Z}{\Delta \Rightarrow Z} \\
\frac{\Gamma(\Delta A) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta A)) \Rightarrow Z}{\Delta \Rightarrow Z}
\end{align*}
\] (A.5)

\[
\begin{align*}
\frac{\sqrt{B}, \Gamma, \sqrt{B} \Rightarrow A}{\text{IR}} & \quad \frac{\Gamma(\sqrt{B}) \Rightarrow \sqrt{B}}{\text{IL}} \\
\frac{\Gamma(\Delta(\sqrt{B}, B, \sqrt{B})) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta B)) \Rightarrow Z}{\Delta \Rightarrow Z} \\
\frac{\Gamma(\Delta B) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta B)) \Rightarrow Z}{\Delta \Rightarrow Z}
\end{align*}
\] (A.6)

\[
\begin{align*}
\frac{\sqrt{B}, \Gamma, \sqrt{B} \Rightarrow A}{\text{IR}} & \quad \frac{\Gamma(\sqrt{B}) \Rightarrow \sqrt{B}}{\text{IL}} \\
\frac{\Gamma(\Delta(\sqrt{B}, B, \sqrt{B})) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta B)) \Rightarrow Z}{\Delta \Rightarrow Z} \\
\frac{\Gamma(\Delta B) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta B)) \Rightarrow Z}{\Delta \Rightarrow Z}
\end{align*}
\] (A.7)

J is like I but has no left rule:

\[
\begin{align*}
\frac{\square A, \sqrt{A}, \sqrt{A} \Rightarrow \sqrt{A}}{\text{id}^2} & \quad \frac{\text{id}^1}{} \\
\frac{\Gamma(\Delta \Rightarrow Y)}{\Gamma(\Delta A) \Rightarrow Y} & \quad \frac{\Gamma(\Delta(\Delta A)) \Rightarrow Y}{\Gamma(\Delta \Rightarrow Y)} \\
\frac{\Delta \Rightarrow Y}{\Delta(\Delta A) \Rightarrow Y} & \quad \frac{\Delta \Rightarrow Y}{\Delta(\Delta A) \Rightarrow Y}
\end{align*}
\] (A.8)

The rules for the discontinuity connectives are as follows:

\[
\begin{align*}
\frac{\sqrt{B}, \Gamma, \sqrt{B} \Rightarrow A}{\text{IR}} & \quad \frac{\Gamma(\sqrt{B}) \Rightarrow \sqrt{B}}{\text{IL}} \\
\frac{\Gamma(\Delta(\sqrt{B}, B, \sqrt{B})) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta B)) \Rightarrow Z}{\Delta \Rightarrow Z} \\
\frac{\Gamma(\Delta B) \Rightarrow Z}{\Delta \Rightarrow Z} & \quad \frac{\Gamma(\Delta(\Delta B)) \Rightarrow Z}{\Delta \Rightarrow Z}
\end{align*}
\] (A.9)
\[
\Gamma(\sqrt{A}) \Rightarrow \sqrt{A} \quad \Delta \Rightarrow B \quad \circ R \\
\Gamma(\Delta) \Rightarrow A \circ B \\
\Gamma(\sqrt{A}, B, \sqrt{A}) \Rightarrow Z \quad \circ L \\
\Gamma(A \circ B) \Rightarrow Z
\]

(A.11)

References


