Meta-Learning methods

Mario Martin partially based on Ricard Gavalda’s slides

UPC - Computer Science Dept.
Outline

Introduction
   Definition

Voting schemes
   Stacking
   Weighted majority
   Bagging and Random Forests
   Boosting
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Voting schemes
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  Boosting
Multiclassifiers, Meta-learners, Ensemble Learners

- Combining several *weak learners* to give a *strong learner*

- A kind of *multiclassifier* systems and *meta-learners*

- *Ensemble* typically applied to a single type of weak learner
  - All built by same algorithm, with different data or parameters

- Lots of what I say applies to multiclassifier systems in general
Why?

1. They achieve higher accuracy in practice
   ▶ We trade computation time for classifier weakness

2. Combine strengths of different classifier builders
   ▶ And we can incorporate domain knowledge into different learners

3. May help avoiding overfitting
   ▶ This is paradoxical because more expressive than weak learners!

4. Good option for online learning in evolving realities

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   - Netflix competition (2009) won by a combination of 107 hybrid classifiers
   - More: Most of the top teams were multi-classifiers
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Combining weak learners

- **Voting**
  - Each weak learner votes, and votes are combined

- **Experts that abstain**
  - A weak learner only counts when it’s expert on this kind of instances
  - Otherwise it abstains (or goes to sleep)
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Voting

How to combine votes?

- Simple majority vote
- Weights depend on errors
  \[ \frac{1}{e_i} \exp(-e_i) \ldots \]
- Weights depend on confidences
- Maximizing diversity
Voting

How to combine votes?
Voting

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- Simple *majority vote*
- Weights depend on *errors* (\(1 - e_i\), \(1/e_i\), \(\exp(-e_i)\), \ldots)
Voting

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- Weights depend on *confidences*
- Maximizing *diversity*
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Stacking (Wolpert 92)

A meta-learner that learns to weight its weak learner

- Dataset with instances \((x, y)\)
- Transform dataset to have instances \((x, c_1(x), \ldots, c_N(x), y)\)
- Train metaclassifier \(M\) with enriched dataset

Often, \(x\) not given to \(M\), just the votes

Often, just linear classifier

Can simulate most other voting schemes
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Weighted majority (Littlestone-Warmuth 92)

initialize classifiers $c_1\ldots c_N$ with weight $w_i = 1/N$ each;

for each example $x$ in sequence do

collect predictions $c_1(x)\ldots c_N(x)$;

$\text{prediction}(x) = \text{sign}\left[w_1*c_1(x)+\ldots+w_N*c_N(x)\right]-1/2$

get true label $y$ for $x$;

for each $i=1\ldots N$, if ($c_i(x) \neq y$) then $w_i = w_i/2$;

renormalize weights to sum 1;

▶ Weights depend exponentially on error
▶ At least as good as best weak learner in time $O(\log N)$
▶ Often much better; more when classifiers are uncorrelated
▶ Good for online prediction and when many classifiers
▶ E.g. when 1 classifier = 1 feature
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for each example x in sequence do
    collect predictions c1(x)...cN(x);
    prediction(x) = sign[ w1*c1(x)+...+wN*cN(x)]-1/2 ]
initialize classifiers $c_1...c_N$ with weight $w_i = 1/N$ each;
for each example $x$ in sequence do
  collect predictions $c_1(x)...c_N(x)$;
  prediction($x$) = $\text{sign}[ w_1c_1(x)+...+w_Nc_N(x)]-1/2$ ]
get true label $y$ for $x$;
initialize classifiers $c_1 \ldots c_N$ with weight $w_i = 1/N$ each;

for each example $x$ in sequence do
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get true label \( y \) for \( x \);
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initialize classifiers $c_1 \ldots c_N$ with weight $w_i = 1/N$ each;
for each example $x$ in sequence do
  collect predictions $c_1(x) \ldots c_N(x)$;
  prediction($x$) = sign[$w_1 \cdot c_1(x) + \ldots + w_N \cdot c_N(x) - 1/2$]
  get true label $y$ for $x$;
  for each $i=1..N$,
    if ($c_i(x) \neq y$) then $w_i = w_i / 2$;
  renormalize weights to sum 1;
initialize classifiers $c_1...c_N$ with weight $w_i = 1/N$ each;
for each example $x$ in sequence do
    collect predictions $c_1(x)...c_N(x)$;
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- Weights depend exponentially on error
- At least as good as best weak learner in time $O(\log N)$
- Often much better; more when classifiers are uncorrelated
- Good for online prediction and when many classifiers
- E.g. when 1 classifier = 1 feature
1. Get a dataset $S$ of $N$ labeled examples on $A$ attributes;
2. Build $N$ bagging replicas of $S$: $S_1, \ldots, S_N$;
   ▶ $S_i =$ draw $N$ samples from $S$ with replacement;
3. Use the $N$ replicas to build $N$ weak learners $C_1, \ldots, C_N$;
4. Predict using majority vote of the $C_i$’s
Example of building training sets:

<table>
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<th>Original:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training Set1:</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Training Set2:</td>
<td>7</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Training Set3:</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Training Set4:</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Specially useful in Decision trees and Perceptrons (high variance or sensibility to training data set)
Bagging II

Way to do majority voting when small samples or only one classifier builder

Improves unstable weak learners (= with high variance)

May degrade stable weak learners (= with low variance)
Random Forests (Breiman 01, Ho 98)

1. Parameters $k$ and $a$;
2. Get a dataset $S$ of $N$ labeled examples on $A$ attributes;
3. Build $k$ bagging replicas of $S$: $S_1, \ldots, S_k$;
4. Use the $k$ replicas to build $k$ random trees $T_1, \ldots, T_k$;
   - At each node split, randomly select $a \leq A$ attributes, and choose best of these $a$;
   - Grow each tree as deep as possible: not pruning!!
5. Predict using majority vote of the $T_i$’s
Weak learner strength vs. weak learner variance

- More attributes $a$ increases strength, overfits more
- More trees $k$ decreases variance, overfits less

Can be shown to be similar to weighted $k$-NN

Top performer in many tasks
Random Forests II

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Boosting I

- Bagging tries to reduce variance of base classifiers by building different bootstrapping datasets
- Boosting tries to actively improve accuracy of weak classifiers
- How? By training a sequence of specialized classified based on previous errors
Adaptively, sequentially, creating classifiers

Classifiers and instances have varying weights

Increase weight of incorrectly classified instances
Boosting II

- Works on top of any *weak learner*. A weak learner is defined as any learning mechanism that works better than chance (accuracy $> 0.5$ when two equally probable classes)
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- Adaptively, sequentially, creating classifiers
- Classifiers and instances have varying weights
- Increase weight of incorrectly classified instances
- Final label as weighting voting of sequence of classifiers
Preliminaries

- Only two classes
- Output: \( y \in \{-1, 1\} \)
- Examples: \( X \)
- Weak Classifier: \( G(X) \)
- Error de training (\( err_{train} \))

\[
err_{train} = \frac{1}{N} \sum_{i=1}^{N} I(y_i \neq G(x_i))
\]
Preliminaries

Weighted Sample → $G_M(x)$

Weighted Sample → $G_3(x)$

Weighted Sample → $G_2(x)$

Training Sample → $G_1(x)$

Weight of each classifier

$G(x) = \text{sign} \left( \sum_{m=1}^{M} \alpha_m G_m(x) \right)$
Adaboost algorithm

Set weight of all examples to $1/n$
Adaboost algorithm

Set weight of all examples to $1/n$
For $t=1:L$

Return classifier: $G(x) = \text{sign} \left( \sum_{t=1}^{L} \alpha_t G_t(x) \right)$
Adaboost algorithm

Set weight of all examples to $1/n$
For $t=1:L$
    $S_t = \text{training set using weights for each example}$
Adaboost algorithm

Set weight of all examples to $1/n$

For $t=1:L$

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Learn $G_t(S_t)$
Adaboost algorithm

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For $t=1:L$
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    Compute $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - err_t}{err_t} \right)$
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For $t=1:L$

$S_t =$ training set using weights for each example

Learn $G_t(S_t)$

Compute $err_t$ for $G_t$

Compute $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \text{err}_t}{\text{err}_t} \right)$

Compute new weights $w_i \leftarrow \frac{w_i}{Z_t} \cdot e^{-[\alpha_t \cdot y_i \cdot G_t(x_i)]}$

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Return classifier: $G(x) = \text{sign} \left( \sum_{t=1}^{L} \alpha_t G_t(x) \right)$
Adaboost algorithm

$$\alpha_m = \frac{1}{2} \ln \left( \frac{1 - err_m}{err_m} \right) > 0$$

$$w_i \leftarrow \frac{w_i}{Z_m} \cdot e^{-[\alpha_m \cdot y_i \cdot G(x_i)]}$$

$$G(x) = \text{sign} \left( \sum_{m=1}^{L} \alpha_m G_m(x) \right)$$
Adaboost algorithm

\[ \alpha_m = \frac{1}{2} \ln \left( \frac{1 - \text{err}_m}{\text{err}_m} \right) > 0 \]

\[ w_i \leftarrow \frac{w_i}{Z_m} \cdot \left\{ \begin{array}{ll} e^{-\alpha_t} & \text{if } y_i = G_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq G_t(x_i) \end{array} \right. \]

\[ G(x) = \text{sign} \left( \sum_{m=1}^{L} \alpha_m G_m(x) \right) \]
We will use *Decision stumps* as the weak learner.

Decision stumps are decision trees pruned to only one level. Good candidates to weak learners: above 0.5 accuracy and high variance.

Two examples of decision stumps.
Simple example

$D_1$
Simple example

\[ \varepsilon_1 = 0.30 \]
\[ \alpha_1 = 0.42 \]
Simple example
Simple example

\[ h_3 \]

\[ \epsilon_3 = 0.14 \]

\[ \alpha_3 = 0.92 \]
\[ H_{\text{final}} = \text{sign}(0.42 + 0.65 + 0.92) \]
Simple example

\[ f = \left( \begin{array}{c} 0.42 \\ +0.65 \\ +0.92 \end{array} \right) / (0.42 + 0.65 + 0.92) \]
Theorem. Suppose that the error of classifier $h_t$ is $1/2 - \gamma_t$, $t = 1..T$. Then the error of the combination $H$ of $h_1, \ldots, h_T$ is at most

$$\exp \left( - \sum_{t=1}^{T} \gamma_t^2 \right)$$

Note: It tends to 0 if we can guarantee $\gamma_i \geq \gamma$ for fixed $\gamma$. 
Boosting vs. Bagging

- Fruitful investigation on how and why they differ
- On average, Boosting provides a larger increase in accuracy than Bagging
- But Boosting fails sometimes (particularly in noisy data)
- while bagging consistently gives an improvement
Possible reasons why this works

1. **Statistical reasons**: We do not rely on one classifier, so we reduce variance.

2. **Computational reasons**: A weak classifier can be stuck in local minima. When starting from different training data sets, we can find better solution.

3. **Representational reasons**: Combination of classifiers return solutions outside the initial set of hypothesis, so they adapt better to the problem.
Possible reasons why this works

All the previous reasons seem to drive us to an overfitting on the training data set.
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However, in practice this is not the case. Not well understood theoretical reasons.
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In practice they work very well, sometimes better that SVMs.