Vickery-Clark-Groves Mechanism

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1. Selling one item
2. VCG mechanism

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- A losing bidder pays nothing and gets $u_i = 0$

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- Revenue equivalence results

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Can this be achieved in other settings?

- Moving from a specific example (1-item auctions) to a more general mechanism design setting.
- Objective: Design the right incentives such that the efficient outcome will be chosen.

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- TV costs $100
- They should decide:
  - Do they want to buy a TV together?
  - If so, how should they share the costs?
- Individual preferences have to be taken into consideration in deciding the efficient outcome.

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  A general design rule is the Vickrey-Clarke-Groves mechanism.

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  Here, again, truthfulness is a dominant strategy.

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  - $p_1 = 100 - 80$.

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  - But, total payment is $20 + 30 < 100$!

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- There isn't much we can do: It can be shown that there is no mechanism that is both efficient and budget balanced.