Outline

I. Introduction

II. Symbolic computation of guards
I. Introduction

The synthesis problem:

Global Guard

Local Guards

Good markings

Very bad markings

Bad markings

I. Introduction

Global Guard = BE

Local Guards
II. Symbolic Computation of guards

Reachability Relation

\[ RR = \{(m, m') | \exists t \in T \text{ s.t. } m[t > m']\} \]

\[ X_{RR}(p, p') = \left( \bigwedge_{\pi_i \in t} (m[p_i] \geq \text{Pre}[p_i, t]) \right) \cdot \left( \bigwedge_{\pi_i \in t \setminus t'} (m'[p_i] = m[p_i] - \text{Pre}[p_i, t]) \right) \cdot \left( \bigwedge_{\pi_i \in t' \setminus t} (m'[p_i] = m[p_i] + \text{Post}[p_i, t]) \right) \cdot \left( \bigwedge_{\pi_i \in t' \cap t} (m'[p_i] = m[p_i] - \text{Pre}[p_i, t] + \text{Post}[p_i, t]) \right) \cdot \left( \bigwedge_{\pi_i \in T \setminus (t \cup t')} (m'[p_i] = m[p_i]) \right) \]

\[ X_{RR}(p, p') = \bigvee_{t \in T} X_{RRt}(p, p') \]

II. Symbolic Computation of guards

Using de reachability relation

\[ X_{RR}(p, p') = \bigvee_{t \in T} X_{RRt}(p, p') \]

Compute the set of reachable markings from \( m_0 \)

\[ X_{m1}(p') = X_{m0}(p') + \exists(p) [X_{m0}(p) \cdot X_{RR}(p, p')] \]
\[ X_{m2}(p') = X_{m1}(p') + \exists(p) [X_{m1}(p) \cdot X_{RR}(p, p')] \]
\[ X_{m3}(p') = X_{m2}(p') + \exists(p) [X_{m2}(p) \cdot X_{RR}(p, p')] \]
\[ \vdots \]
\[ \text{until } X_{m(k+1)}(p') = X_{mk}(p') \]
Application to the control of a DES

- F is a set of states to be forbidden (e.g. states that inevitably lead to a deadlock)
- Compute the boundary set of F
  - $X_A = X_{RS} \setminus X_F$
  - $X_{RR_{fr}}(q,p) = X_A(q) \cdot X_{RR}(q,p) \cdot X_F(p)$
  - $X_{F_{fr}} = \exists(q) X_{RR_{fr}}(q,p)$

**II. Symbolic Computation of guards**

Application to the control of a DES

Profit don’t care conditions

Minimize $X_{F_{fr}}$ using the set of don’t cares: $\neg (X_A + X_{F_{fr}})$

After the simplification:

$$X_{F_{fr}} = a \cdot d + a \cdot e + b \cdot d$$

RS = A U F
II. Symbolic Computation of guards

Application to the control of a DES

Exercise 1. Compute a guard per transition from the information of the boundary set.

Exercise 2. Compute set of cutting places from the information of the boundary set.

Computation of the transition guards

For each transition \( t \):

- \( X_{\text{RR}_{fr}}(q, p) = X_{\text{RR}_{fr}}(q, p) \wedge_p \in t \ p \)
- The guard: \( X_G(t)(q) = \neg (\exists(p) X_{\text{RR}_{fr}}(q, p)) \)
- Minimize with the set of don’t cares:

\[
\neg (X_A \wedge_p \in t \ p)
\]

Simplified guards:

- \( X_{G1} = \neg e \cdot \neg d \)
- \( X_{G2} = \neg d \)
- \( X_{G3} = \text{True} \)
- \( X_{G4} = \text{True} \)
- \( X_{G5} = \neg a \cdot \neg b \)
- \( X_{G6} = \neg a \)
- \( X_{G7} = \text{True} \)
- \( X_{G8} = \text{True} \)
II. Symbolic Computation of guards

From the simplified $X_{Fr}$ we can obtain a set of inequalities from which the places to be added are derived.

\[ X_{Fr} = ad + ae + bd \]

Linear system:
- \( a + d > 1 \)
- \( a + e > 1 \)
- \( b + d > 1 \)

Places to be added:
- \( p_1, p_2, p_3 \)

Bibliography