Outline

I. Net subclasses
II. State Machines and Marked Graphs
III. Well-formedness and rank theorems
IV. Modular Petri Nets
V. The problem of net subclasses
I. Net subclasses

- Net subclasses are defined at the syntactical level, i.e. imposing constraints to the interplaying of conflicts and synchronizations
  - Modelling capabilities are reduced
  - Analysis is easier

- Some syntactical subclasses (ordinary nets)
  - State machine: for all $t \in T$: $|\cdot t| = |t\cdot| = 1$
  - Marked graphs: for all $p \in P$: $|\cdot p| = |p\cdot| = 1$
  - Free Choice:
    - for every $p, q \in P$: $p\cdot \cap q\cdot = \emptyset$ or $p\cdot = q\cdot$
    - for every $t, u \in T$: $\cdot t \cap \cdot u = \emptyset$ or $\cdot t = \cdot u$
  - Asymmetric Choice, or simple:
    - for every $p, q \in P$: $p\cdot \cap q\cdot = \emptyset$ or $p\cdot \supseteq q\cdot$ or $q\cdot \supseteq p\cdot$
II. State Machines & Marked Graphs

Essential properties [Murata 1989]

- State Machines
- Marked Graphs
- Acyclic Petri Nets
- ........

III. Well-formedness and rank Th.

Well-formedness (SB&SL)

- A net $\mathcal{N}$ is SB $\iff (\mathcal{N}, M_0)$ bounded for every $M_0$
- A net $\mathcal{N}$ is SL $\iff (\mathcal{N}, M_0)$ live for some $M_0$

A general necessary condition:

$\mathcal{N}$ well-formed $\Rightarrow \mathcal{N}$ Cv&Ct

- $\mathcal{N}$ Cv: $\exists Y > 0$ such that $Y \cdot C = 0$
- $\mathcal{N}$ Ct: $\exists X > 0$ such that $C \cdot X = 0$

Goal:

- Necessary or sufficient conditions for general nets
- Necessary and sufficient conditions for subclasses
  
  ...in polynomial time
III. Well-formedness and rank Th.

- t, t’ in Choice Relation:
  \( t = t' \) OR \( t \cap t' \neq \emptyset \)
  Not transitive

- Coupled Conflict Relation:
  Transitive closure of Choice Relation.
  Quotient set: \( \zeta \)

- t, t’ in Equal Conflict Relation:
  \( t = t' \) OR \( \text{Pre}[P,t] = \text{Pre}[P,t'] \neq 0 \)
  Quotient set: \( \xi \)

\[ N \text{ well-formed } \Rightarrow N \text{ Cv and Ct and rank}(C) \leq |\xi| - 1 \]

* A regulation circuit on an Equal Conflict e of a well-formed net doesn’t spoil well-formedness, and increments the rank in \(|e| - 1\). Regulating every Equal Conflict:

\[ |T| - 1 \geq \text{rank}(C') = \text{rank}(C) + \sum_{e \in E}(|e| - 1) = \text{rank}(C) + |T| - |\xi| \]

\[ N \text{ Cv and Ct and rank}(C) = |\xi| - 1 \Rightarrow N \text{ well-formed} \]
III. Well-formedness and rank Th.

Let $N$ be a Cv and Ct net

- Impossible (against Cv&Ct)
- Not well-formed
- Well-formed
- Impossible (against Cv&Ct)

Impossible (against Cv&Ct) $\Rightarrow$ $|\xi| - 1$
Not well-formed $\Rightarrow$ $\min(|P|, |T|) - 1$
Well-formed $\Rightarrow$ $|\xi| - 1$
Impossible (against Cv&Ct) $\Rightarrow$ $|\zeta| - 1$

$\xi = \{[t_1, t_2], [t_3], [t_4]\}$
$\zeta = \{[t_1, t_3], [t_2], [t_4]\}$

Both nets are Cv&Ct

$\text{rank}(C) = 3 > |\xi| - 1 = 2$
$\text{rank}(C) = 2 = |\zeta| - 1$
III. Well-formedness and rank Th.

The EC Rank Theorem

\[ N \text{ well-formed } \iff N \text{ Cv&Ct and } \text{rank}(C) = |\xi| - 1 = |\zeta| - 1 \]

- Polynomial time complexity

\[ (N, M_0) \text{ B&L } \iff N \text{ well-formed and every P-component live} \]

- Polynomial time complexity when ordinary

IV. Modular Petri Nets

Deterministic Systems of Sequential Processes

A DSSP is a P/T net system composed of:
- A set of Sequential Processes
  - 1-safe strongly connected State Machines \((N_i, M_{0i})\)
- A set of buffers
  - Buffers are output-private
  - Buffers respect internal choices of the SP
    - Reisig (1982): Buffers I/O private; no weights
    - Souissi et al. (1988): Buffers I/O private
Well-formedness of DSSP

\[
N \text{ well-formed} \iff N \text{ Cv& Ct and rank}(C) = |\xi| - 1
\]

- Polynomial time complexity
IV. Modular Petri Nets

- an *admissible* initial marking
- composition of production plans with resources via fusion of resource places
IV. Modular Petri Nets

- Four classical necessary conditions:
  - mutual exclusion
  - hold and wait
  - no preemption
  - circular wait

\[ \text{Siphon } S \subseteq P \]

\[ S \subseteq S^* \]

Characterisation based on the net structure

**Theorem.** Let \( \langle N, m_0 \rangle, N=\langle P_0 \cup P_s \cup P_R, T, C \rangle \), be a marked S\(^4\)PR. The net is non-live if, and only if, there exists a marking \( m \in RS<N, m_0 > \), with \( m[P_s] > 0 \), and a siphon \( D \) such that the firing of each \( m\)-process-enabled transition is prevented by a set of resource places belonging to \( D \).
V. The problem of net subclasses

Implicit places

\[ m[p_2] = m[p_a] + m[p_b], \quad \forall m \in RS(N, m_0) \]
\[ m[p_3] = m[p_b] + m[p_c], \quad \forall m \in RS(N, m_0) \]
\[ m[p_5] = m[p_a] + m[p_c] + m[p_4], \quad \forall m \in RS(N, m_0) \]

\[ m[p_a] = m[p_1] + m[p_2] + m[p_5], \quad \forall m \in RS(N, m_0) \]
\[ m[p_b] = m[p_1] + m[p_3] + m[p_5], \quad \forall m \in RS(N, m_0) \]
\[ m[p_c] = m[p_1] + m[p_2] + m[p_3] + m[p_4], \quad \forall m \in RS(N, m_0) \]

Given a Petri Net and an initial marking

Canonical Form?