L08- Transformation of Petri Nets

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Outline

I. Introduction
II. A Basic Kit of Reduction Rules
III. Macroplace Reduction Rule
IV. Implicit Places
V. Something magic in Petri Nets
I. Introduction

- **Net system transformations vs. Net system reductions**
- **RULE:**
  - Structure precondition + Marking precondition
  - Structure change + Marking change
- **APPLICATION:**
  - *if* preconditions are true *then* make changes
- **PROBLEM:**
  - There exist non-reducible nets for a given reduction kit
  - Tradeoff: kit reduction power versus kit complexity
- **REMARK:** For net subclasses (e.g. Live and Bounded Free Choice Systems), there exist complete reduction kits

II. A basic kit of reduction rules

**Anatomy of a reduction rule**

- Structural conditions
- Marking conditions
- List of preserved properties
II. A basic kit of reduction rules

Applied rules

RI. Fusion of series places

RA2. Fusion of series transitions

II. A basic kit of reduction rules

RA1. Fusion of series places

RA2. Fusion of series transitions

a)

b)

c)

d)
IV. Implicit Places

The critical section problem for n processes

Semaphore: \( \text{MUTEX} := 1 \)

Process \( P_i \)

\[
\text{repeat} \\
\quad \text{wait}(\text{MUTEX}); \\
\quad \text{Critical section} \\
\quad \text{signal}(\text{MUTEX}); \\
\quad \text{Private work} \\
\text{until false;}
\]

II. A basic kit of reduction rules

Applied rules

RC1  
RA2

From the reduced net we conclude: The original one is live and bounded
III. Macroplace Reduction Rule

**Macroplace reduction rule** - Substitution of a P-graph (N is a P-graph iff for all \( t \in T \), \(|t\bullet| = |\bullet t| = 1\)) by one single place.

**Some terminology**
- Let \( N' = (P', T', F') \) be a subnet of \( N = (P, T, F) \) [i.e. \( F' = F \cap ((P' \times T') \cup (T' \times P')) \)]
- A place \( p' \in P' \) is a **way-in place** of \( N' \) iff \( p \cap (T \setminus T') \neq \emptyset \), where the dot notation refers to \( N \). [Way-in places are those that can be used to “enter” into the subnet]
- A place \( p' \in P' \) is a **way-out place** of \( N' \) iff \( p' \cap (T' \setminus T) \neq \emptyset \), where the dot notation refers to \( N \). [Way-out places are those through which we can “get out” of the subnet]

**Definition (Subnet reducible to a place).**
Let \( N' \) be a subnet of \( N \). \( N' = (P', T', F') \) is reducible to a place if:

1) \( N' \) is a P-graph containing at least one transition and for all \( t \in T' \): \(|t' \cap P'| \leq 1\) and \(|\bullet t \cap P'| \leq 1\)

2) For every \( p' \in P' \), there exists at least an \( F' \)-path from a way-in place of \( N' \) to \( p' \).

3) For every \( p' \in P' \) and every way-out place \( p_o' \) of \( N' \), there exists an \( F' \)-path from \( p' \) to \( p_o' \).
III. Macroplace Reduction Rule

**Definition (Substitution of a net by a place).**

Let \( \langle N = (P,T,F), m_0 \rangle \) be a system and \( N' = (P',T',F') \) a subnet of \( N \) reducible to a place \( \pi \). The reduced net \( N_r = (P_r,T_r,F_r) \), where

- \( P_r = (P \setminus P') \cup \{ \pi \} \)
- \( T_r = (T \setminus T') \)
- \( F_r = (F \cap ((P_r \times T_r) \cup (T_r \times P_r))) \cup F_\pi \), where

  - \((t,\pi) \in F_\pi \) iff there exists \((t,p') \in F\) with \(p' \in P'\)
  - \((\pi,t) \in F_\pi \) iff there exists \((p',t) \in F\) with \(p' \in P'\)

is a macroplace reduction of \( N \), and \( \pi \) is the macroplace that replace \( N' \). The system \( \langle N_r, m_r \rangle \) where \( N_r \) is the reduction of \( N \) and \( m_r \) is given by:

- \( m_r[p] = m_0[p] \) if \( p \neq \pi \)
- \( m_r[\pi] = \sum_{p \in P'} m_0[p] \)

is called a macroplace reduction of \( \langle N, m_0 \rangle \)

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**Observation**

The application of the Macroplace Reduction Rule requires to find the subnets of a net that can be reduced to a place.

Efficient polynomial algorithms exist for this purpose.

It consists of removing first all transitions with more than one input or more than one output arc. This splits the net into one or more P-graphs.

Then simple recursive procedures are applied to each connected subnet to check conditions (b) and (c) of the definition of reducible subnets to a place.

**Observation**

The reduction rule preserves liveness & the bound of the system
III. Macroplace Reduction Rule
III. Macroplace Reduction Rule

IV. Implicit Places

**Definition (Implicit Place, IP).**

A place p is *implicit* in \(<N,m_o>\) if never is the unique to constraint the occurrence of its output transitions

- Therefore, removing implicit places *does not change the set of occurrence sequences*

- The implicit place reduction rule *preserve liveness and synchronic properties* (lead, distance, slack, fairness...)
IV. Implicit Places

**Definition (Structurally Implicit Place, SIP).**
A place $p$ is *structurally implicit* in $N$ if for any initial marking, the initial marking of $p$ can be chosen such that $p$ becomes implicit.

**Property (Algebraic characterization of SIPs)**
A place $p$ is structurally implicit iff there exists $y \geq 0$, $y[p] = 0$ and $y \cdot C \leq C[p]$

**Property (Initial markings of SIPs to be IPs)**
If $p$ is structurally implicit and $m_o[p] \leq y \cdot m_o$ then $p$ is implicit.

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![Diagram](image)

a) $p \in$ IP but $p \notin$ SIP    

b) $p \in$ IP and $p \in$ SIP
Example

- p is a structurally implicit place
- p is implicit
- **Implying places**: p₄ and p₅

\[ m[p] = m[p₄] + m[p₅] \]
for all \( m \in RS(N,m₀) \)

### IV. Implicit Places

- **Marking Implicit places**
  \[ \exists y \geq 0 \text{, } y[p] = 0 \text{ and } y \cdot C = C[p,T] \]
  \[ m[p] = y \cdot m + k \]

- **Firing Implicit places**
  \[ \exists y \geq 0 \text{, } y[p] = 0 \text{ and } y \cdot C \not\subseteq C[p,T] \]
  \[ m[p] \geq y \cdot m + k \]

The place p in the figure is firing implicit place (and unbounded!)
Sets of implicit places

\( p_1, p_2, \text{ and } p_3 \) are implicit places simultaneously

\( p_4, p_5, \text{ and } p_6 \) are implicit places simultaneously

IV. Implicit Places

**Property (Initial markings of SIPs to be IPs)**

If \( p \) is a structurally implicit place and \( m_0[p] \geq \max(0,z) \) then \( p \) is implicit

\[
\text{LPPi} = \begin{array}{l}
\text{minimize} \quad y \cdot m_o + \mu \\
\text{subject to} \quad y \cdot C \leq C[p,T] \\
\quad y[p] = 0 \\
\quad y \cdot \text{Pre}[P,t] + \mu \geq \text{Pre}[p,t], \text{ for all } t \in p^* \\
\quad y \geq 0
\end{array}
\]

If \( \mu < 0 \) in the optimal solution then the variable \( m[p] \) is not a non-extremal variable (i.e. the linear equation of \( m[p] \) is not redundant in the net state equation) !!!!!
IV. Implicit Places

The place $p_9$ is implicit


Optimal solution of LPPi

$y = (0 0 1 1 1 0 1 0 0)$

$\mu = -1$

Initial marking to be implicit

$m_o[p_9] = y \cdot m_o + \mu = 1 - 1 = 0$

$m_o[p_9] = 0$

V. Something magic in Petri nets

Transformation/Reduction of nets

Implicit places

$m[p_2] = m[p_a] + m[p_b]$, $\forall m \in RS(N,m_o)$

$m[p_3] = m[p_b] + m[p_c]$, $\forall m \in RS(N,m_o)$

$m[p_3] = m[p_a] + m[p_c] + m[p_4]$, $\forall m \in RS(N,m_o)$

$m[p_a] = m[p_1] + m[p_2] + m[p_3]$, $\forall m \in RS(N,m_o)$

$m[p_b] = m[p_1] + m[p_2] + m[p_3]$, $\forall m \in RS(N,m_o)$

$m[p_c] = m[p_1] + m[p_2] + m[p_3] + m[p_4]$, $\forall m \in RS(N,m_o)$
Given a Petri Net and an initial marking

Canonical Form?