Soft Comput (2010) 14:421–434
DOI 10.1007/s00500-009-0455-8

FOCUS

LSP suitability maps

Jozo J. Dujmović · Guy De Tré · Nico Van de Weghe

Published online: 5 June 2009

Abstract In this paper, we propose the concept of logically aggregated geographic suitability maps (LSP-maps). The goal of LSP-maps is to provide specialized maps of the suitability degree of a selected geographic region for a specific purpose. There is a wide spectrum of purposes which include suitability for industrial development, agriculture, housing, education, recreation, etc. Our goals are to specify the main concepts of LSP-maps development and to identify some of the potential application areas. Our approach is based on soft computing with partial truth and graded logic functions within the framework of the LSP method.

1 Introduction

Geographic maps are traditionally defined as the distribution of selected scalar indicators in the two-dimensional space. For complex planning and decision making each (*X*, *Y*) location may be characterized by an array of attributes $(a_1, a_2, \ldots, a_n)$ that are used as inputs for decision models. Such attributes may describe physical characteristics of terrain (slope, altitude, material, distance from major roads, distance from green areas, distance from lakes, etc.), available infrastructure (supply of water, supply of electrical energy, sewage system, telecommunications, transport systems, etc.), urban characteristics (distance from major schools, shopping areas, entertainment, sport facilities, hospitals, the density of population, etc.), legal status (private property, governmental property, areas reserved for special activities), economic development (local industries, businesses, employability), pollution (air, water, noise), etc. All these attributes affect the overall suitability of a specific location for a selected type of use. In a general case the degree of suitability depends on a variety of logic conditions that evaluators specify using reasoning techniques that are typical for soft computing.

The LSP-map is defined as a spatial distribution of the overall degree of suitability for a specific use. The most frequent decision problems that need suitability maps are problems of planning and development, and problems of environmental protection. The most common types of development include commercial, industrial, residential, and military development, as well as the use of selected regions as the farmland or the forestland. In the protection area, we are interested in the use of selected regions as natural areas that may be ecologically important (for wildlife habitat and biodiversity) or important as natural heritage areas. More specific examples of suitability for development include the suitability for construction of industrial objects, homes, hospitals, schools, recreation areas, entertainment centers, sport facilities, shopping centers, airports, etc. In all cases, decision makers are interested to evaluate and compare locations or regions

J. J. Dujmović (✉)
Department of Computer Science,
San Francisco State University, 1600 Holloway Ave,
San Francisco, CA 94132, USA
e-mail: jozo@sfsu.edu

G. De Tré
Department of Telecommunications and Information Processing,
Ghent University, St.-Pietersnieuwstraat 41,
9000 Ghent, Belgium
e-mail: Guy.DeTre@UGent.be

N. Van de Weghe
Department of Geography, Ghent University,
Krijgslaan 281 (S8), 9000 Ghent, Belgium
e-mail: Nico.VandeWeghe@UGent.be

from the standpoint of their suitability for a selected use. The degree of suitability E is a soft computing logical function of n attributes and we assume that its range is normalized: $E = G(a_1, a_2, \ldots, a_n) \in [0, 1]$. The value 0 denotes an unsuitable location and the value 1 (or 100%) denotes the maximum level of suitability.

Our concept is similar to the suitability maps proposed in (Joerin et al. 2001) and (Wood and Dragićević 2007). However, the land-use suitability maps (Joerin et al. 2001) are based on outranking methods and marine protection maps (Wood and Dragićević 2007) are derived using simple additive scoring. Such approaches do not support flexible logic conditions that we consider fundamental for justifiable decision making. Similar to (Joerin et al. 2001) and (Chakhar and Mousseaua 2007), our approach is a step toward dynamic generation of specialized maps based on multicriteria decision models.

The predecessors of LSP-maps are composed using map algebra. Map algebra is a set based algebra for manipulating geographic data (Tomlin 1990), or some of its generalizations (e.g., Camara et al. 2005) or extensions [e.g., with the temporal dimension (Frank 2005; Mennis et al. 2005)]. Notwithstanding that map algebra is recognized as one of the most dominant frameworks to handle GIS-based raster data (Longley et al. 2001), alternatives (e.g., Haklay 2004; Chakhar and Mousseaua 2007) have been proposed, all having their pros and contras.

While traditional maps are always produced having in mind the needs and interest of specific users, there is a clear need for specialized composite indicators that can be dynamically generated in a flexible way from geographic databases to provide information necessary for advanced public and professional decision making related to urban planning, industrial development, corporate planning, etc. In particular, there is a need for soft computing suitability maps that show suitability indicators based on flexible suitability criteria that include sophisticated logic conditions.

The purpose of this paper is to propose a method for designing LSP-maps using the LSP method for evaluating suitability. We first present the concept of LSP-maps and then expand this concept to cases where suitability includes financial components. The details of developing a suitability map are presented using a numerical case study of evaluating suitability for urban expansion. At the end, we discuss the issues of providing accurate input attributes data that affect the reliability of LSP-maps.

2 The Concept of LSP-maps and their use in decision making

The proposed technique for creating LSP-maps is summarized in Fig. 1. The investigated area is divided in an orthogonal grid of square cells of size h where X, Y denote the coordinates of the center of a specific cell. Each analyzed cell is characterized by an array of n cell attributes $(a_1(X, Y), a_2(X, Y), \ldots, a_n(X, Y))$. The attributes are indicators that affect the ability of the analyzed cell to support some desired activity. For simplicity, the array of attributes can be denoted $(a_1, a_2, \ldots, a_n)$, and we assume that each attribute is a function of coordinates X, Y.

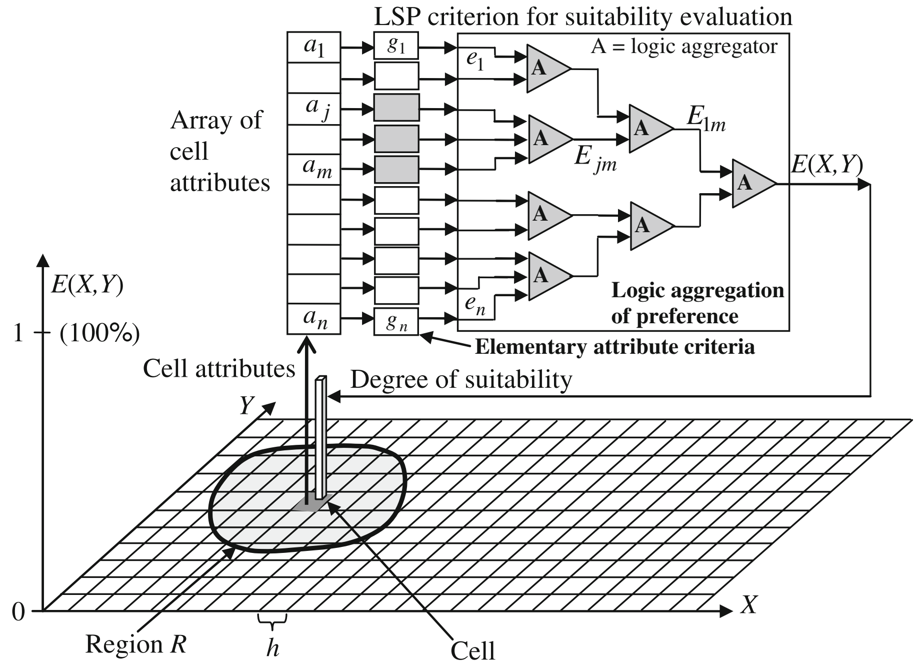
The array of attributes provides inputs for the quantitative evaluation process based on the LSP method (Dujmović 1987, 2007; Dujmović and Nagashima 2006). After defining a complete and nonredundant list of input attributes, the next step in this process is to provide elementary attribute criteria for each component of the array of attributes. The elementary criteria are functions $g_i : R \rightarrow [0, 1], \;\; i = 1, \ldots, n$. The value $e_i = g_i(a_i)$ is called the attribute (or elementary) preference. The attribute preference denotes the degree to which the value a_i satisfies a specific requirement that reflects the selected type of evaluation.

The final step in the organization of the LSP criterion function is the development of the preference aggregation structure that logically aggregates all attribute preferences and generates the resulting overall preference that is the degree of suitability for a specific purpose: $E(X, Y) = \lambda(e_1, \ldots, e_n) = \lambda(g_1(a_1), \ldots, g_n(a_n)) \in [0, 1]$. The aggregation process can include a variety of logic conditions that are modeled using the generalized conjunction/disjunction (GCD) function (Dujmović 2008) and more complex compound aggregators in continuous preference logic (Dujmović 2007). The stepwise logic aggregation of n inputs, shown in Fig. 1, provides expressive power that exceeds other approaches to making suitability maps. Details of creating LSP criteria for LSP-maps and the logic aggregation process are presented in subsequent sections.

The value $E(X, Y)$ reflects the suitability of a given X, Y cell, and the distribution defined by $E(X, Y)$, $X_{\min} \leq X \leq X_{\max}$, $Y_{\min} \leq Y \leq Y_{\max}$ represents the desired LSP-map. Of course, our goal is to identify areas of high suitability and to use them in rational and justifiable decision making.

The overall suitability can be computed in various situations, as illustrated in Fig. 2. The most frequent application is the comparison of suitability of several discrete locations (e.g., the suitability of locations A, B, and D for building an airport). The suitability can be computed along a line or a curve (e.g., a pipeline path suitability, or the suitability of building a rest area along the highway L). Finally, the suitability can also be computed inside a closed region R or along its contour line (e.g., the suitability of a coastal line of an island for building new hotels, or the suitability of the area of the island for food production or for urban development).

Fig. 1 The concept of LSP-maps



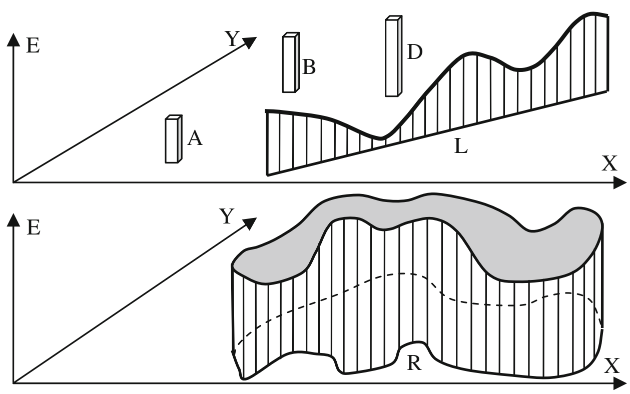


Fig. 2 Suitability in discrete points *A*, *B*, *D*, along a path *L*, and in a closed region *R*

The average suitability along the line/path/contour $Y = L(X)$ consisting of line segments $\ell_1, \ell_2, \ldots$ and inside the region R consisting of cells with areas $A_1, A_2, \ldots$ can be computed by averaging $E(X, Y)$ as follows:

$$\bar{E} = \frac{\int_L E(X, Y)\mathrm{d}\ell}{\int_L \mathrm{d}\ell} \approx \frac{\sum_i E(X_i, L(X_i))\ell_i}{\sum_i \ell_i},$$
$$\bar{E} = \frac{\iint_R E(X, Y)\mathrm{dXdY}}{\iint_R \mathrm{d}X\mathrm{d}Y} \approx \frac{\sum_{A_i \in R} E(X_i, Y_i)A_i}{\sum_{A_i \in R} A_i}$$

The mean suitability $\bar{E}$ is a useful indicator only if the distribution $E(X, Y)$satisfies some basic acceptability criteria, primarily a sufficient smoothness and a low variability. For example, if $E(X, Y)$in a region R shows discontinuities or large variations, this can prevent some applications regardless of the value of $\bar{E}$.

LSP-suitability maps are a convenient tool for solving some of the environmental decision problems. Suppose that we have a region R that can be used for conservation purposes as a habitat for K species that could live in the region. The region R can be contiguous, or it can be a set of smaller regions. We also assume that the realization of protection of wildlife in region R is related to the overall cost C that aggregates the costs of purchasing, adapting, monitoring and protecting R. Let $E_i(X, Y), i = 1, \ldots, K$ be the suitability distributions of the region R for K relevant species. In addition, suppose that the analyzed species have relative priority degrees $P_i, i = 1, \ldots, K$. The nominal priority degree is 1, and various species (depending on how endangered they are) have priorities that can be above or below the nominal level.

The factors that determine the value (V) of region R for conservation purposes include the suitability of region, its size, and its ability to protect those species that have high priorities. Thus, it is reasonable to define the following total value of region R:

$$V = \sum_{i=1}^{K} P_i \iint_R E_i(X, Y)\mathrm{d}X\mathrm{d}Y$$

If we have N competitive regions then their aggregated quality score can be computed as follows:

$$Q_j = V_j / C_j, \quad V_j = \sum_{i=1}^{K_j} P_i \iint_R E_i(X, Y)\mathrm{d}X\mathrm{d}Y,$$
$$j = 1, \ldots, N$$

The optimum allocation of available conservation funds would be to first acquire the region with the highest Q_j

value, followed with the second highest value, and so on. More sophisticated techniques for computing Q_j as a function of V_j and C_j are described in the next section.

It is important to note that LSP-maps can be dynamically generated from the geographical database of attributes. They are flexible because the formal logic and semantic parameters that evaluate the suitability of cells can be interactively modified and adjusted by the user. By modifying the parameters the user can generate a sequence of maps that answer a variety of "what-if" questions. These answers are the primary purpose of LSP-maps.

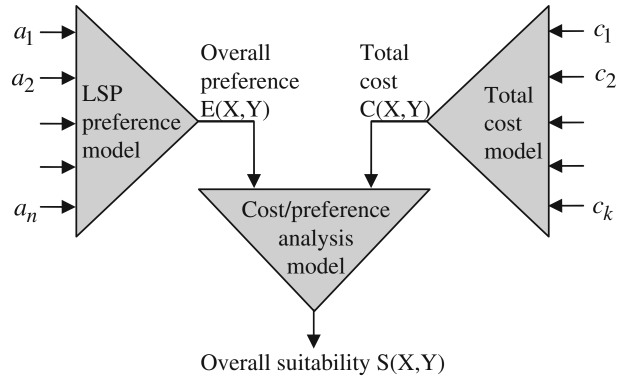


Fig. 3 Computation of the overall suitability using a cost/preference model

3 Financial components of LSP-maps

Generally, the computation of $E(X, Y)$ should include all relevant inputs, and in many cases suitability attributes may also include financial indicators. For example, in the case of evaluating the suitability for urban development, we frequently need to take into account the cost of terrain, the cost of providing transport and energy infrastructure, the cost of building objects, the cost of workforce, the cost of financing, etc. Therefore, various cost components could be interpreted as input attributes and integrated in the array$(a_1, a_2, \ldots, a_n)$. However, we do not suggest the integrated approach. Following are the basic reasons why we suggest keeping cost analysis as a separate subproblem when designing suitability maps:

(a) Elementary criteria for various cost components should transform a cost component into elementary preference, and substantial efforts are needed to establish acceptable individual cost criteria for each cost component of a specific suitability problem.
(b) Cost components regularly create additive compensation patterns, i.e., we usually add all costs and can compensate higher costs of some components by lower costs of other components. However, the logic aggregation of preferences is usually based on complex and nonlinear logic aggregation functions with many adjustable parameters, causing cost compensation patterns that are not easily justifiable.
(c) The overall preference score for a specific use of geographic locations is paid with the total cost of attaining such a goal. Therefore, by the very nature of this problem, the overall preference relates to the total cost, and the overall suitability should be obtained by aggregating the total cost and overall preference.

If the cost analysis model is separated from the preference score model, then the computation of the overall suitability can be realized by the cost/preference analysis shown in Fig. 3. The overall preference $E(X, Y)$ is computed from strictly nonfinancial suitability attributes $(a_1, a_2, \ldots, a_n)$. The total cost indicator $C(X, Y)$ computed using a total cost model that aggregates individual cost components and related financial conditions $(c_1, c_2, \ldots, c_k)$. In a trivial case, the cost model can be the simple sum of all cost components. In more complex cases $C(X, Y)$ can aggregate all related expenses and earnings and take into account the dynamic of cash flow expressed by its present value.

Cost/preference analysis models for computing the overall suitability $S(X, Y)$ as a function of $E(X, Y)$ and $C(X, Y)$can be designed in a variety of ways. Generally, we would like to simultaneously have a high preference and a low cost. In the simplest case this requirement could be expressed as $S(X, Y) = E(X, Y)/C(X, Y)$. This simplistic model assumes that the cost and the preference are equally important, and that we know nothing about the ranges of acceptable values of the cost and the preference. In addition, this version of $S(X, Y)$ has a nonstandard range that is sometimes difficult to interpret.

More precise models with higher expressive power can be based on compound indicators of inexpensiveness and usefulness. The simplest inexpensiveness indicator can be defined as follows:

$$P(X, Y) = \frac{\min_R C(X, Y)}{C(X, Y)}, \quad 0 < P \leq 1$$

This indicator attains the maximum value 1 in the points of minimum cost, and has lower values in all other points. The minimum cost, however, can still be above the highest affordable value. Therefore, if evaluators believe that the cost must not reach or exceed a given maximum value $C_{\max}$, then a normalized inexpensiveness indicator can be defined as follows:

$$P(X, Y) = \max\left(0, \frac{C_{\max} - C(X, Y)}{C_{\max}}\right), \quad 0 \leq P(X, Y) \leq 1$$

In this case, if $C(X, Y)$ reaches or exceeds the maximum value $C_{\max}$, then $P(X, Y)$ takes the minimum value 0. The maximum value $P(X, Y) = 1$is obtained only if $C(X, Y) = 0$. If this is a too stringent requirement then we can expand the inexpensiveness indicator as follows:

$$P(X, Y) = \max\left[0,\ \min\left(1, \frac{C_{\max} - C(X, Y)}{C_{\max} - C_{\min}}\right)\right],\quad 0 \le P(X, Y) \le 1$$

In this case, the inexpensiveness indicator attains the maximum value $P(X, Y) = 1$ when $C(X, Y) \le C_{\min}$. Consequently, by selecting appropriate values of the parameters $C_{\max}$ and $C_{\min}$ we can precisely adjust desired features of the inexpensiveness criterion.

The overall preference $E(X, Y)$ expresses the usefulness of a geographic location for selected purpose, without taking cost components into consideration. We can use $E(X, Y)$ and aggregate it with inexpensiveness $P(X, Y)$ to compute the overall suitability $S(X, Y)$. For example, the suitability criterion $S(X, Y) = \sqrt{E(X, Y)P(X, Y)}$ reflects the request to simultaneously have high values of both preference and inexpensiveness, assuming that the relative importance of preference equals the relative importance of inexpensiveness. Of course, this is not always the case and we frequently need criteria that are more expressive and more flexible. In such cases, it is more convenient to apply overall usefulness indicators that are similar to the inexpensiveness indicators.

The simplest usefulness indicator is the following relative preference:

$$U(X, Y) = \frac{E(X, Y)}{\max_R E(X, Y)},\quad 0 \le U(X, Y) \le 1$$

This indicator is helpful in cases where the absolute level of preference is not very important, and in regions with high overall preference. However, we frequently face the situation where there is a threshold minimum preference $E_{\min}$ and if $E(X, Y) \le E_{\min}$ then such geographic locations are considered unacceptable for the selected purpose. In such cases, it is convenient to use the following overall usefulness indicator:

$$U(X, Y) = \max\left(0, \frac{E(X, Y) - E_{\min}}{1 - E_{\min}}\right),\quad 0 \le U(X, Y) \le 1$$

Using this indicator we create the range of acceptable preferences $]E_{\min}, 1]$ and all geographic regions outside the acceptable range are considered equally unsuitable.

After selecting appropriate usefulness and inexpensiveness indicators, we can aggregate them and compute the overall suitability. The corresponding aggregator must have two fundamental properties: (1) an adjustable level of simultaneity in satisfying the usefulness and the inexpensiveness criteria, and (2) the ability to model different levels of relative importance of usefulness and inexpensiveness. An aggregator that satisfies these criteria is a GCD (Dujmović 2008) that can be implemented using the following weighted power mean (WPM):

$$S(X, Y) = \left[W_u(U(X, Y))^r + W_p(P(X, Y))^r\right]^{1/r},$$
$$0 \le W_u \le 1,\quad 0 \le W_p \le 1,\quad W_u + W_p = 1,$$
$$-\infty \le r \le +\infty$$

The weights W_u and W_p are used to express the relative importance of usefulness and inexpensiveness. In some cases, it is more important to have high usefulness than to attain a low cost. Of course, there are also the opposite situations where the inexpensiveness is the major decision factor. Therefore, the equal importance of usefulness and inexpensiveness is a frequent special case, but generally the overall suitability depends on usefulness and inexpensiveness in a way that is not symmetrical. Extreme cases where $W_u = 1 - W_p = 0$ or $W_u = 1 - W_p = 1$ are infrequent but possible. These are cases where the total suitability depends only on financial feasibility or only on usefulness. For example, a map of "scenic value" might only have the U component.

Geographic locations that simultaneously attain high levels of both usefulness and inexpensiveness are the most attractive. As a natural consequence, the aggregator of usefulness and inexpensiveness is regularly some form of partial conjunction. The necessary level of simultaneity in satisfying usefulness and inexpensiveness criteria is measured using the global andness indicator (Dujmović 2007), and can be adjusted using the parameter r. The most useful values are $r = 1$ (andness = 50%), $r = 1/2$ (andness = 58%), $r = 0$ (andness = 67%), $r = -1$ (andness = 77%) and $r = -2$ (andness = 83%). If $r \le 0$ then both usefulness and inexpensiveness become mandatory (if one of them is not satisfied then $S(X, Y) = 0$). For example, if we want to make a suitability map where usefulness and inexpensiveness are mandatory, but the andness is as low as possible, the usefulness must be above $E_{\min}$, the cost is expected to be in the range $[C_{\min}, C_{\max}[$, and the usefulness is considered two times more important than the inexpensiveness, then the suitability should be computed as follows:

$$S(X, Y) = \left\{\max\left(0, \frac{E(X, Y) - E_{\min}}{1 - E_{\min}}\right)\right\}^{\frac{2}{3}} \times \left\{\max\left[0, \min\left(1, \frac{C_{\max} - C(X, Y)}{C_{\max} - C_{\min}}\right)\right]\right\}^{\frac{1}{3}}$$

The computation of$E(X, Y)$ is based on the LSP method. In subsequent sections, our goal is to show details of creating LSP suitability maps using a case study of suitability for urban development. The LSP method includes three fundamental steps: (1) the development of a system attribute tree, (2) the definition of elementary criteria (one criterion for each attribute), and (3) the development of a logic aggregation structure that will be used to aggregate all elementary preferences and generate the overall preference $E(X, Y)$.

4 The suitability attribute tree

The suitability for urban expansion is one of the frequent and nontrivial suitability problems. The first step in the LSP suitability map development consists of creating a list of suitability attributes. This process consists of hierarchical decomposition of overall usefulness into components, where in each step of decomposition the complexity of components decreases. The decomposition eventually yields components that are sufficiently simple and cannot be further decomposed. These are suitability attributes. A sample system attribute tree is shown in Fig. 4. Initially, we decompose the usefulness for urban expansion into three basic components: the terrain and environment, the location and accessibility, and the population and employment opportunities. Each group is then decomposed into subgroups; e.g., in the first group we identify terrain properties and the environment properties. The terrain properties include the slope, altitude, and terrain orientation. These components are sufficiently simple and cannot be further decomposed; consequently, these are suitability attributes. The resulting attribute tree includes 11 attributes. Of course, the tree can be more detailed and include any number of attributes, depending on the desired precision of the suitability map.

Some of the attributes are considered mandatory, i.e., if they are not satisfied then the overall usefulness for urban expansion is considered unacceptable and rated zero. Mandatory attributes in Fig. 4 are denoted by (+). On the other hand, there are attributes that are in our example considered nonmandatory and denoted by (−). If a nonmandatory requirement is not satisfied that will not cause the zero overall usefulness and the rejection of the analyzed location. For example, while the appropriate slope and altitude are considered mandatory requirements, a good orientation of the new urban complex is considered desirable, but it is not mandatory. Similarly, good environment is highly desirable but not necessary: if other conditions are satisfied new urban areas can be built despite the absence of green areas and lakes. Finally, the proximity to an airport is also considered nonmandatory. However, good ground transportation is considered mandatory.

Mandatory and nonmandatory attributes are the simplest examples of logic conditions that are present in all areas of evaluation. Additional logic conditions include the adjustable levels of simultaneity or replaceability of attributes, as well as more complex compound logic relationships, presented in Sect. 6.

1 Terrain and environment (+)
- 11 Terrain properties (+)
  - 111 Slope (+)
  - 112 Altitude (+)
  - 113 Orientation of terrain (-)
- 12 Environment properties (-)
  - 121 Proximity of forests or major green areas (-)
  - 122 Proximity of a lake/river (-)

2 Location and accessibility (+)
- 21 Ground transportation (+)
  - 211 Proximity of an interstate highway (+)
  - 212 Proximity of a regional highway (+)
  - 213 Proximity of an intercity railroad station
- 22 Proximity of an international airport (-)

3 Population and employment opportunities (+)
- 31 Density of population (+)
- 32 Proximity to employment opportunities (+)

Fig. 4 System attribute tree with mandatory (+) and nonmandatory (−) components

In addition to logic relationships it is also necessary to specify the levels of relative importance (weights) of all components in all decomposition groups. This process becomes difficult (and usually also imprecise) whenever the number of components in a group becomes large. Our experience yields a suggestion to use up to five elements in a group. In such small groups it is possible to accurately estimate relative importance without using sophisticated computational methods and corresponding software tools.

5 Elementary criteria

The second step in the design of LSP suitability maps is to specify elementary criteria, i.e., individual requirements for all attributes. The requirements are specified as functions that show the level of satisfaction with each value of the attribute. The level of satisfaction is called the elementary preference and it belongs to interval [0,1] (or [0, 100%]). The elementary preference 0 reflects an unacceptable value of the input attribute, and the value 1 (or 100%) reflects a value that completely satisfies evaluation requirements. The elementary preferences between 0 and 1 reflect partial satisfaction of evaluation requirements. Sample elementary attribute criteria for the urban expansion suitability map are shown in Fig. 5. We used the following three characteristic forms of criteria:

(a) Monotonically increasing form (criterion #113)
(b) Monotonically decreasing form (criteria #111, 112, 121, 122, 213, 22, 32)
(c) Trapezoid filtering form (criteria #211, 212, and 31)

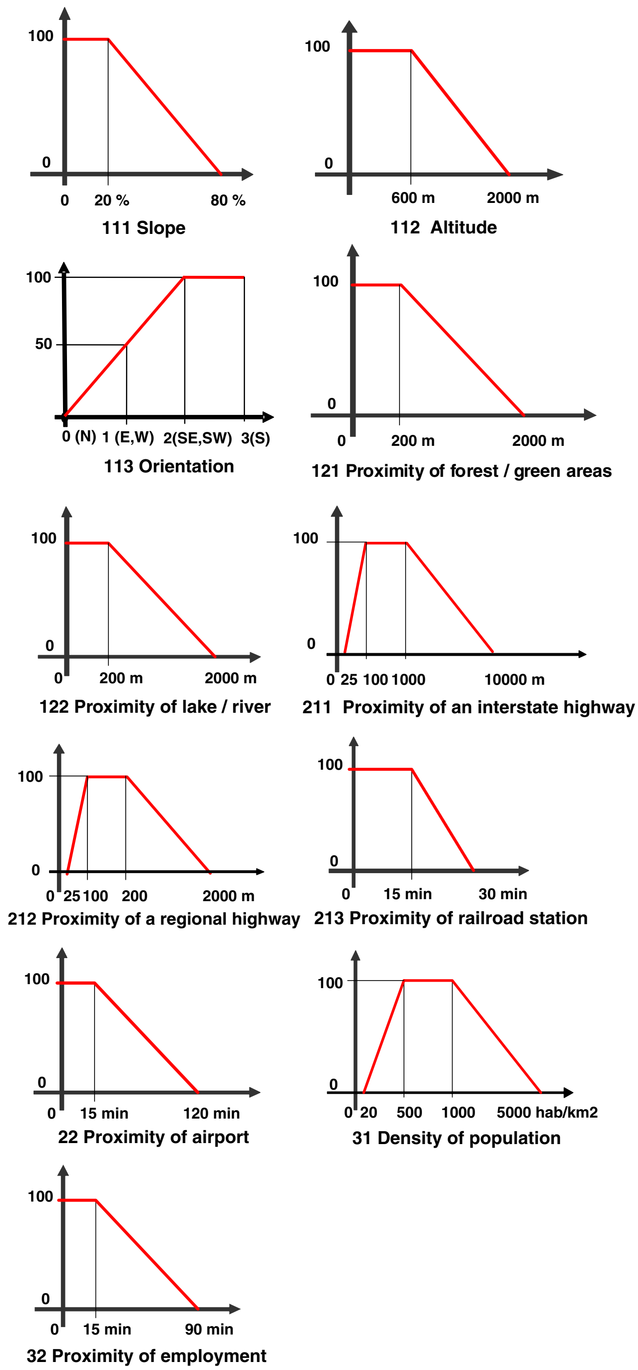


Fig. 5 Sample elementary criteria

Elementary criteria are usually based on piecewise linear (polygonal) approximations of functions: we define a set of justifiable breakpoints and use linear interpolation between them. This approach yields a good combination of simplicity and accuracy.

All elementary criteria reflect a desired evaluation standpoint and can be verbally interpreted. For example, the criterion #22 specifies that the access time to an airport of 15 min or less is ideal, while the access time of 120 min is unacceptable. For access times between 15 and 120 min we use a simple linear interpolation:

$$e_{22} = 100\frac{120 - a}{105}[\%].$$

So, if a = 36 min then the corresponding elementary preference score is $e_{22} = 80\%$. In other words, the access time of 36 min satisfies 80% of our requirements. This criterion might be criticized from the standpoint that the closest neighborhood of an airport could be congested, polluted, and unsafe, and therefore it cannot be considered a perfectly suitable location for urban expansion. If we want to discourage the urban development in the vicinity of an airport then the criterion #22 should be modified to have a trapezoid form, similar to the criterion #212. Obviously, elementary criteria can reflect a spectrum of different standpoints, and the validity of an LSP-map fundamentally depends on the proper justification of elementary criteria.

The criterion #212 is an example of trapezoid elementary criterion. It specifies that it is desired that an urban complex is located in the proximity of a regional highway, but not too close to it. More precisely, the proposed elementary criterion considers that an ideal distance from the regional highway is from 100 to 200 m. If the distance is greater than 2,000 m or less than 25 m that is considered unacceptable. All other criteria from Fig. 5 can be verbally interpreted in a similar way.

6 Logic aggregation of preferences

The third step in the design of LSP suitability maps is the organization of the preference aggregation structure. The goal of this step is to aggregate all elementary preferences and to generate the overall preference following the pattern of the aggregation tree from leaves toward the root. Each aggregator computes the usefulness of a group as a function of the usefulness of group components. Individual components are not equally important, and sometimes we need their simultaneous satisfaction, sometimes they can replace each other, sometimes they are all mandatory, sometimes some of them are mandatory and others are only desired, etc. In other words, each aggregation function must be able to express logic and semantic relationships between components in a justifiable way that is derived from the knowledge of a domain expert.

Aggregators that satisfy the majority of evaluation requirements must be derived from observable properties

Table 1 Classification of fundamental CPL aggregators (Dujmović 2008)

Aggregation operators in Continuous Preference Logic (CPL)	Generalized Conjunction/ Disjunction (GCD) – the basic CPL aggregator	Full disjunction (D)	
		Partial disjunction (PD)	Hard partial disjunction (HPD)
			Soft partial disjunction (SPD)
		Neutral aggregator	Arithmetic mean (A)
		Partial conjunction (PC)	Soft partial conjunction (SPC)
			Hard partial conjunction (HPC)
		Full conjunction (C)	
	Compound aggregators	Simple partial absorption	Disjunctive partial absorption (DPA)
			Conjunctive partial absorption (CPA)
		Nested partial absorption	Sufficient/Desired/Optional (SDO)
			Mandatory/Desired/Optional (MDO)
		Partial equivalence, partial implication, etc.	

of human evaluation reasoning. This approach was used in continuous preference logic (Dujmović JJ 2007) where we identified fundamental aggregators that are presented in Table 1. Seven of these aggregators are special cases of the GCD (Dujmović 2008), and four compound aggregators are frequently used partial absorptions obtained by superposition of various special cases of GCD.

Two basic special cases of GCD are the partial conjunction and the partial disjunction. Partial conjunction is a model of simultaneity, and the partial disjunction is a model of replaceability. The partial conjunction (symbolically denoted $x_1\Delta\ldots\Delta x_n$) is similar to the traditional full conjunction $x_1\wedge\ldots\wedge x_n$, and the partial disjunction (symbolically denoted $x_1\nabla\ldots\nabla x_n$) is similar to the traditional full disjunction $x_1\vee\ldots\vee x_n$ $(x_i\in[0,1],$ $i=1,\ldots,n)$. The degree of similarity between any form of GCD and the full conjunction is called andness (α), and the degree of similarity between any form of GCD and the full disjunction is called orness (ω). To define andness and orness we first note that all forms of GCD (symbolically denoted $x_1\diamondsuit\ldots\diamondsuit x_n$) are located in the range from the full conjunction to the full disjunction. That also holds for their mean values:

$$\overline{x_1\diamondsuit\ldots\diamondsuit x_n}=\int_0^1 dx_1\ldots\int_0^1(x_1\diamondsuit\ldots\diamondsuit x_n)dx_n,$$

$$\overline{x_1\wedge\ldots\wedge x_n}\le\overline{x_1\diamondsuit\ldots\diamondsuit x_n}\le\overline{x_1\vee\ldots\vee x_n}$$

$$\overline{x_1\wedge\ldots\wedge x_n}=\int_0^1 dx_1\ldots\int_0^1(x_1\wedge\ldots\wedge x_n)dx_n=\frac{1}{n+1},$$

$$\overline{x_1\vee\ldots\vee x_n}=\int_0^1 dx_1\ldots\int_0^1(x_1\vee\ldots\vee x_n)dx_n=\frac{n}{n+1}$$

So, we can define global andness and orness as the position of GCD between conjunction and disjunction, as follows:

$$\alpha=\frac{\overline{x_1\vee\ldots\vee x_n}-\overline{x_1\diamondsuit\ldots\diamondsuit x_n}}{\overline{x_1\vee\ldots\vee x_n}-\overline{x_1\wedge\ldots\wedge x_n}}$$

$$=\frac{n-(n+1)\overline{x_1\diamondsuit\ldots\diamondsuit x_n}}{n-1}=1-\omega,\quad 0\le\alpha\le1$$

$$\omega=\frac{\overline{x_1\diamondsuit\ldots\diamondsuit x_n}-\overline{x_1\wedge\ldots\wedge x_n}}{\overline{x_1\vee\ldots\vee x_n}-\overline{x_1\wedge\ldots\wedge x_n}}$$

$$=\frac{(n+1)\overline{x_1\diamondsuit\ldots\diamondsuit x_n}-1}{n-1}=1-\alpha,\quad 0\le\omega\le1$$

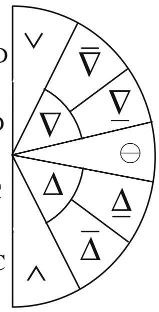
If we want to use GCD as an aggregator, we must first select a desired degree of andness/orness which determines the desired strength or partial conjunction/disjunction. Then, we look for a suitable function that can implement GCD. A survey of seven special cases of GCD and their properties is presented in Table 2.

The border between the SPC and HPC is located at the threshold andness α_θ, and the border between SPD and HPD is located at the threshold orness ω_θ. Generally, $1/2<\alpha_\theta<1$, $1/2<\omega_\theta<1$, $\alpha_\theta\ne\omega_\theta$. Both α_θ and ω_θ depend on the selected implementation of GCD.

Both C and HPC are models of high simultaneity and mandatory requirements. All inputs must be (partially) satisfied, and therefore they reflect mandatory requirements. If any input in an aggregated group of preferences is 0, the output is going to be 0. In the case of C the andness is constant and has the extreme value 1 (100%), and in the case of HPC the andness is lower (but usually above 66%) and adjustable.

SPC is also a model of simultaneity, but its (adjustable) level of simultaneity is lower than in the case of HPC (the SPC andness is usually between 50 and 66%). No input is

Table 2 Seven special cases of GCD and their characteristic properties

GCD symbols and aggregator inequality	Aggregator	Characteristic properties
$\Diamond$ { $\vee =$ D, $\bar{\bar{\nabla}} =$ HPD, $\nabla =$ PD, $\underline{\nabla} =$ SPD, $\ominus =$ A, $\underline{\Delta} =$ SPC, $\Delta =$ PC, $\bar{\bar{\Delta}} =$ HPC, $\wedge =$ C }  *Aggregator inequality:* $x_1 \wedge \ldots \wedge x_n < x_1 \bar{\bar{\Delta}} \ldots \bar{\bar{\Delta}} x_n <$ $< x_1 \underline{\Delta} \ldots \underline{\Delta} x_n < x_1 \ominus \ldots \ominus x_n <$ $< x_1 \underline{\nabla} \ldots \underline{\nabla} x_n < x_1 \bar{\bar{\nabla}} \ldots \bar{\bar{\nabla}} x_n <$ $< x_1 \vee \ldots \vee x_n$ $x_i \neq x_j, \quad i \neq j,$ $i \in \{1,\ldots,n\}, \; i \in \{1,\ldots,n\}$	**D** (full disjunction)	$\alpha=0,\ \omega=1$ $x_1 \vee \ldots \vee x_n = 1, \;\; x_i = 1, i \in \{1,\ldots,n\}$
	HPD (Hard partial disjunction)	$0 < \alpha \le 1 - \omega_\theta,\ \omega_\theta \le \omega < 1$ $x_1 \bar{\bar{\nabla}} \ldots \bar{\bar{\nabla}} x_n = 1,\ x_i = 1, i \in \{1,\ldots,n\}$
	SPD (Soft partial disjunction)	$1 - \omega_\theta < \alpha < 0.5,\ 0.5 < \omega < \omega_\theta$ $x_1 \ominus \ldots \ominus x_n \le x_1 \underline{\nabla} \ldots \underline{\nabla} x_n < 1,$ $x_i < 1, \quad i \in \{1,\ldots,n\}$
	A (neutrality)	$\alpha=0.5,\ \omega=0.5$
	SPC (Soft partial conjunction)	$0.5 < \alpha < \alpha_\theta, 1 - \alpha_\theta < \omega < 0.5$ $0 < x_1 \underline{\Delta} \ldots \underline{\Delta} x_n \le x_1 \ominus \ldots \ominus x_n$ $x_i > 0, \quad i \in \{1,\ldots,n\}$
	HPC (Hard partial conjunction)	$\alpha_\theta \le \alpha < 1,\ 0 < \omega \le 1 - \alpha_\theta$ $x_1 \bar{\bar{\Delta}} \ldots \bar{\bar{\Delta}} x_n = 0,\ \ x_i = 0, i \in \{1,\ldots,n\}$
	C (full conjunction)	$\alpha=1,\ \omega=0$ $x_1 \wedge \ldots \wedge x_n = 0, \;\; x_i = 0, i \in \{1,\ldots,n\}$

mandatory. A single nonzero input is sufficient to produce a (small) nonzero output.

D, HPD, and SPD are models of replaceability symmetrical to C, HPC, and SPC. The orness distribution of HPD and SPD is similar to the andness distribution of HPC and SPC. D and HPD are models of high replaceability and sufficient requirements. If only one input is completely satisfied (i.e., its preference is 1), that is sufficient to completely satisfy the whole group and the values of other inputs are insignificant. Each input can fully compensate (replace) all remaining inputs. In the case of D the orness is constant and has the maximum value 1, and in the case of HPD the orness is lower and adjustable.

SPD is also a model of replaceability, but its (adjustable) level of replaceability is lower than in the case of HPD. No input is sufficient to completely satisfy the whole group, but any nonzero input is sufficient to produce a nonzero output.

The neutrality aggregator A (arithmetic mean) provides a perfect logic balance between simultaneity and replaceability. The andness and the orness are equal (½ or 50%). Thus, the logic interpretation of the arithmetic mean is that it represents a 50–50 mix of conjunctive and disjunctive properties, that is explicitly visible in the case of two inputs: $x_1 \ominus x_2 = (x_1 + x_2)/2 = [(x_1 \wedge x_2) + (x_1 \vee x_2)]/2$. For any number of inputs, all inputs are desired and each of them can partially compensate the insufficient quality of any other of them. No input is mandatory and no input is able to fully compensate the absence of all other inputs. In other words, the arithmetic mean simultaneously, with medium intensity, satisfies two contradictory requests: (1) to simultaneously have all good inputs, and (2) that each input has a moderate ability to replace any other input.

The arithmetic mean is located right in the middle of GCD aggregators but we cannot use it as a single best representative of all of them. The central location of the arithmetic mean is not sufficient to give credibility to additive scoring methods. Indeed, it is difficult to find an evaluation problem without mandatory requirements, or without the need to model various levels of simultaneity and/or replaceability, or without compound asymmetric logic aggregators that frequently aggregate mandatory and optional inputs. These features are ubiquitous and indispensable components of human evaluation reasoning. Unfortunately, these features are not supported by the arithmetic mean. Therefore, in the majority of evaluation problems the additive scoring represents a dangerous oversimplification because it is inconsistent with observable properties of human evaluation reasoning.

GCD can be implemented in various ways (Dujmović 2008). If we want all seven special cases of GCD we can use the following implementation based on WPM:

$$x_1 \Diamond \ldots \Diamond x_n = \begin{cases} x_1 \vee \ldots \vee x_n = \max(x_1, \ldots, x_n), & r = +\infty \\ x_1 \bar{\nabla} \ldots \bar{\nabla} x_n = 1 - [W_1(1-x_1)^{2-r} + \cdots + W_n(1-x_1)^{2-r}]^{1/(2-r)}, & 2 \le r < +\infty \\ x_1 \underline{\nabla} \ldots \underline{\nabla} x_n = 1 - [W_1(1-x_1)^{2-r} + \cdots + W_n(1-x_1)^{2-r}]^{1/(2-r)}, & 1 < r < 2 \\ x_1 \ominus \ldots \ominus x_n = W_1 x_1 + \cdots + W_n x_n, & (r = 1) \\ x_1 \underline{\Delta} \ldots \underline{\Delta} x_n = (W_1 x_1^r + \cdots + W_n x_n^r)^{1/r}, & 0 < r < 1 \\ x_1 \bar{\Delta} \ldots \bar{\Delta} x_n = (W_1 x_1^r + \cdots + W_n x_n^r)^{1/r}, & -\infty < r \le 0 \\ x_1 \wedge \ldots \wedge x_n = \min(x_1, \ldots, x_n), & r = -\infty \end{cases}$$

$$0 < W_i < 1, \quad i = 1, \ldots, n, \quad W_1 + \cdots + W_n = 1$$

Weights are normalized and used to specify a desired relative importance of inputs. Note that weights affect the mean value of an aggregator. Consequently, the computation of andness and orness is always performed with equal weights ($W_1 = \cdots = W_n = 1/n$). This implementation of GCD is selected so that the partial conjunction and the partial disjunction satisfy De Morgan's laws:

$$x_1 \Delta \ldots \Delta x_n = 1 - (1-x_1) \nabla \ldots \nabla (1-x_n)$$
$$x_1 \nabla \ldots \nabla x_n = 1 - (1-x_1) \Delta \ldots \Delta (1-x_n)$$

In these formulas, we assume that both PC and PD can be either soft or hard and that the andness of PC equals the orness of PD. In this case, both the threshold andness and the threshold orness correspond to the geometric mean; for WPM with equal weights we have $\lim_{r \to 0}(x_1^r/n + \cdots + x_n^r/n)^{1/r} = (x_1 \ldots x_n)^{1/n}$, yielding the following values of α_θ and ω_θ :

$$r = 0, \quad x_1 \bar{\Delta} \ldots \bar{\Delta} x_n = \exp[(\ln x_1 + \cdots + \ln x_n)/n]$$
$$\overline{\exp[(\ln x_1 + \cdots + \ln x_n)/n]} = \left(\frac{n}{n+1}\right)^n$$
$$\alpha_\theta = \frac{n - n^n/(n+1)^{n-1}}{n-1}, \quad \omega_\theta = \frac{n - n^n/(n+1)^{n-1}}{n-1}$$

sufficient requirements, and it is less frequently used. If we do not need HPD and want to have SPD in the full range of orness from 0.5 to 1, then we can use the following implementation of GCD based directly on WPM:

$$x_1 \Diamond \ldots \Diamond x_n = \begin{cases} x_1 \vee \ldots \vee x_n = \max(x_1, \ldots, x_n), & r = +\infty \\ x_1 \underline{\nabla} \ldots \underline{\nabla} x_n = (W_1 x_1^r + \cdots + W_n x_n^r)^{1/r}, & 1 < r < +\infty \\ x_1 \ominus \ldots \ominus x_n = W_1 x_1 + \cdots + W_n x_n, & (r = 1) \\ x_1 \underline{\Delta} \ldots \underline{\Delta} x_n = (W_1 x_1^r + \cdots + W_n x_n^r)^{1/r}, & 0 < r < 1 \\ x_1 \bar{\Delta} \ldots \bar{\Delta} x_n = (W_1 x_1^r + \cdots + W_n x_n^r)^{1/r}, & -\infty < r \le 0 \\ x_1 \wedge \ldots \wedge x_n = \min(x_1, \ldots, x_n), & r = -\infty \end{cases}$$

This is an asymmetric GCD function that slightly deviates from De Morgan's laws, but it is convenient in evaluation practice because it offers HPC, SPC and a wide range of SPD without infrequently used HPD. The value of exponent r can be computed from a desired value of orness, and for this form of GCD function we can use the following numeric approximation (Dujmović JJ 2007):

$$r = \frac{0.25 + 1.6016(\omega - 1/2) + 1.0509(\omega - 1/2)^2 + 2.1631(\omega - 1/2)^3 - 3.3896(\omega - 1/2)^4}{\omega(1-\omega)}$$

In the case of two, three, and four variables $\alpha_\theta = \omega_\theta = 2/3,\ 21/32,\ 244/375$; so, the andness and orness threshold for the analyzed GCD are lower than 67%.

The HPC is modeling mandatory requirements, and is very frequently used. As opposed to that, HPD is modeling

The WPM-based GCD is frequently used and its special cases for andness/orness steps of 1/16 are presented in Table 3.

The selection of basic GCD aggregators is based on answering questions according to the following five steps:

Table 3 17 special cases of GCD and their andness, orness, and symbolic notation

α	1	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	0
ω	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
GCD	C	C++	C+	C+-	CA	C-+	C-	C--	A	D--	D-	D-+	DA	D+-	D+	D++	D
$\Diamond$	$\wedge$	$\bar{\Delta}$	$\bar{\Delta}$	$\bar{\Delta}$	$\bar{\Delta}$	$\bar{\Delta}$	$\underline{\Delta}$	$\underline{\Delta}$	$\ominus$	$\underline{\nabla}$	$\underline{\nabla}$	$\underline{\nabla}$	$\underline{\nabla}$	$\underline{\nabla}$	$\underline{\nabla}$	$\underline{\nabla}$	$\vee$

1. Do we want to model GCD or a compound aggregator?
2. In the case of GCD, do we want simultaneity, or replaceability, or neutrality?
3. In the case of simultaneity/replaceability, do we want it to be hard (supporting mandatory/sufficient inputs) or soft (no mandatory/sufficient inputs)?
4. What is the most appropriate level of andness/orness? For example, in the case of SPC select between C− and C−; in the case of HPC select from C−+, CA, C+−, C+, and C++.
5. What is the ranking of inputs according to their relative importance? Select the appropriate level of relative importance of each input as a normalized weight; the sum of all weights of each GCD aggregator must be 100%.

These aggregators are used in the aggregation structure shown in Fig. 6. Each circle has a reference number and denotes the WPM aggregator $e_{\mathrm{out}} = (W_1 e_1^r + \cdots + W_k e_k^r)^{1/r}$; input lines denote weights $W_1, \ldots, W_k$, and exponents r for aggregators A, C−, C−+, CA, and C+ are 1, 0.619, −0.148, −0.72, and −3.51, respectively (Dujmović and Nagashima 2006). For example, in the final step of aggregation we use the aggregator C+ (HPC at the andness level of 87.5%) and aggregate "terrain and environment", "location and access" and "population and employment" with weights 30, 35, and 35%, respectively. This aggregator can be verbally interpreted as follows:

- We require high simultaneity in satisfaction of the three major inputs
- We consider that "location and access" and "population and employment" are equally important for urban expansion
- Terrain and environment are considered slightly less important than "location and access" and "population and employment"

All aggregators reflect specific opinions about the most appropriate level of simultaneity and the most appropriate degrees of relative importance. Consequently, the selection of aggregators and their parameters always requires expert knowledge and includes reasoning with linguistic variables. Both the usefulness criteria and the role of financial attributes depend on expertly defined objectives. The goal of the LSP method is to help in organizing and quantifying expert knowledge and providing justifiable decisions at the level of complexity that is far beyond the limits inherent to intuitive decision making. Such goal cannot be achieved unless the preference aggregation process is fully compatible with

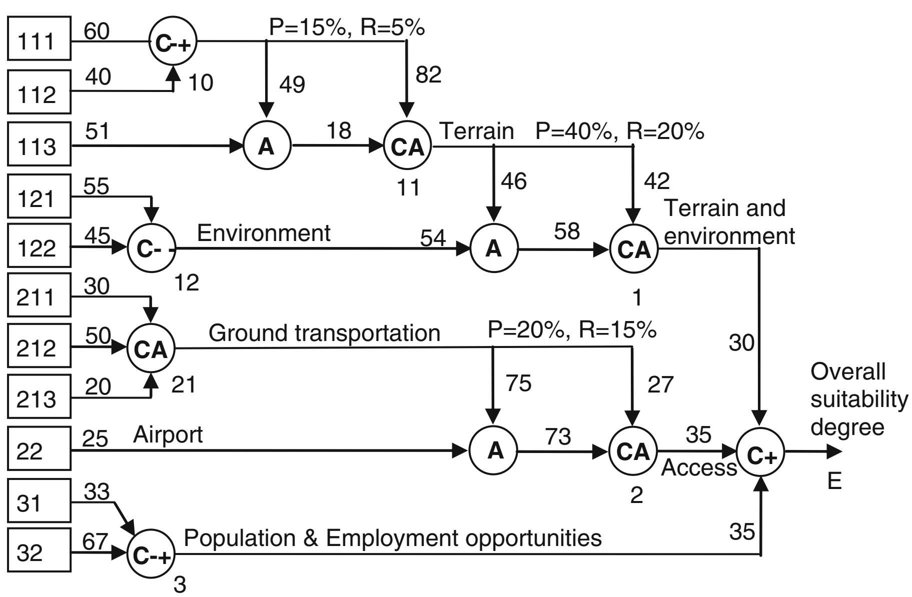


Fig. 6 The aggregation of preferences and the computation of the overall suitability

observable properties of human evaluation reasoning. This is why a flexible logic aggregation of preferences is indispensable in evaluation models.

The aggregation structure shown in Fig. 6 includes mostly conjunctive aggregators that reflect requirements for simultaneous satisfaction of input criteria. However, basic GCD aggregators can be used to create compound aggregators. Three aggregators (identified in Fig. 6 by block numbers 11, 1, and 2) are asymmetrical conjunctive partial absorptions that aggregate mandatory and desired inputs. For example, the CPA aggregator #1 is used for aggregating preferences for "terrain" and "environment." The CPA is a combination of a mandatory input (terrain) and a desired (but not mandatory) input (environment). The mandatory input must be at least partially satisfied: if it is not satisfied (its preference is 0) then the output is 0 regardless the value of the desired input. In other words, the quality of terrain is considered a mandatory requirement: if the terrain in unacceptable there cannot be urban expansion in such an area. On the other hand, the quality of environment is desirable (or optional), but not mandatory: if the terrain is sufficiently suitable for urban development, but forests, lakes and rivers are not close, then our criterion will not reject such locations, but nevertheless consider them (to a controllably reduced extent) suitable for urban development.

If the mandatory input is partially (or completely) satisfied, and the desired input is not satisfied (0), this is going to reduce the value of output preference for an average P% below the level of the mandatory input. The parameter P is called the penalty. If the mandatory input is partially satisfied, and the desired input is completely satisfied (1), this is going to increase the value of the output preference for an average R% above the level of the mandatory input. The parameter R is called the reward.

The penalty and reward determine the fundamental relationships of mandatory and desired/optional inputs of the CPA aggregator. Therefore, the simplest way to select an appropriate CPA aggregator is to select desired values of the penalty and reward. However, the CPA aggregators shown in Fig. 6 have four parameters: two independent weights and two GCD aggregators (A and CA). Usually the A aggregator satisfies all applications and the remaining three parameters can be determined from the desired P/R combination using either the P/R tables (Dujmović 1979) or specialized software tools (Dujmović 1991). In the case of aggregator #1, we use $P = 40\%$ and $R = 20\%$. These are rather large variations that reflect a relatively high significance of the nonmandatory quality of environment in the terrain and environment group. As opposed to that, the CPA #11 uses $P = 15\%$ and $R = 5\%$, reflecting the opinion that the orientation of terrain has a relatively low significance in the mandatory terrain group.

For selecting other compound aggregators (DPA, SDO, MDO) the procedures are similar to the presented procedure for CPA (Dujmović and Nagashima 2006; Dujmović JJ 2007).

7 Sample evaluation and comparison of three locations

For simplicity, let us compare the suitability for urban development of three locations: $L_1 = (X_1, Y_1)$, $L_2 = (X_2, Y_2)$,and $L_3 = (X_3, Y_3)$. Their attributes are shown in Table 4. These locations are selected as typical for three areas that are candidates for urban expansion. Because of differences in the available infrastructure, the cost of building in location L_2 is 30% more expensive than building in location L_1, and building in location L_3 is 20% more expensive than building in location L_1. The problem is to find which location is the most suitable for urban expansion, and the basic results are shown in Table 5.

The most convenient location is L_2, because it satisfies almost 93% of the usefulness requirements. The location L_3 is second, it satisfies 73% of the requirements. The least suitable location is L_1, it satisfies only 52% of the usefulness requirements for urban expansion. If the importance of high usefulness is the same as the importance of low cost then we can compare the locations using the E/C ratio, and in such a case the location L_2 is still the most suitable regardless the highest cost. The E/C ratio can be normalized so that the best option (L_2) is rated 100%. This is done in Table 5 and the overall suitability of L_2 is approximately 15% higher than the suitability of the second best option L_3.

The presented example shows a suitability criterion that incorporates elementary attribute criteria and a number of logic conditions: adjustable andness and orness, adjustable relative importance, symmetric and asymmetric (mandatory/desired) logic conditions. Such criteria can be used for a variety of experiments with other decision requirements.

Table 4 Input attributes and costs for the three competitive locations

Loc	111	112	113	121	122	211	212	213	22	31	32	C
L_1	40	1,200	2	3,000	250	1,500	50	20	100	555	30	1
L_2	18	400	1	150	500	1,100	300	20	20	800	20	1.3
L_3	35	700	0	1,600	700	1,700	400	15	35	1,500	35	1.2

Table 5 Resulting usefulness and suitability degrees

Loc	10	11	12	21	1	2	3	E (%)	E/C
L_1	62.7	65.5	26.8	48.5	51.7	42.9	86	51.75	51.75 (72.5%)
L_2	100	94.4	92.4	88.6	93.8	89.8	95.5	92.83	71.41 (100%)
L_3	81.6	69.7	42	92	60.3	89.9	77.7	72.63	60.52 (84.8%)

8 Data availability and reliability problems

LSP-maps are based on the assumption that accurate values are available for all attributes in the array $(a_1(X,Y), a_2(X,Y),\ldots,a_n(X,Y))$ and for each point (X,Y) in the analyzed region. There are applications where this assumption holds, but there are also real life applications where this is not the case. Information sources might be imperfect, containing inaccurate, incomplete, or even inconsistent data. For some attributes, like distances from given map objects, we would be able to directly compute a reliable attribute value (on condition that the map is reliable). In a general case, however, we must be prepared to face situations where the necessary attribute values are not available. Several types of unavailability have been identified in (Motro 1995), including the incompleteness of data, the lack of sufficiently accurate data, and the non-existent data.

8.1 Handling the incompleteness of data

If an attribute value $a(X,Y)$ for a given point (X,Y) is missing, but values are available for relevant neighboring points that are close enough, then we might be able to derive an approximate value by aggregating the corresponding attribute values of these neighboring points. The problem of deriving a reliable value can thus be decomposed in three subproblems: search for relevant neighboring points, determine whether these are close enough or not, and interpolate the values.

To search for relevant neighboring points, a triangular irregular network (TIN) (Rigaux et al. 2002) is constructed with the points for which data are available as vertices. The well-known Delaunay triangulation method (Delaunay 1934) is used for this purpose. It is then straightforward to determine the triangle in which the point (X,Y) is located; the relevant neighboring points are the vertices of this triangle. After the triangle containing the point (X,Y) is identified the user must decide whether the accuracy of the applied interpolation method is satisfactory. This must be done in a general way and not for each point separately. In the case of unacceptable accuracy, we consider data to be unknown.

In cases where we are interested in interpolation of geologic data it is convenient to use a family of nonlinear least squares estimation algorithms developed in geostatistics, primarily various forms of kriging (Goovaerts 1997; Wackernagel 1995).

8.2 Handling the lack of sufficiently accurate data

In the case of unknown data, we may apply one of the following three approaches: (1) develop a modified suitability criterion excluding unknown data, (2) perform the analysis replacing the unknown value by a range of appropriate values or a distribution, and (3) develop a method to modify the preference aggregation structure in an automatic way.

8.3 The case of nonexistent data

Another potential type of unavailable data is the case where data is not available because it does not exist. This means that the corresponding criterion is not applicable for the analyzed location. This is a sufficient indication that the existing suitability criterion must be redesigned.

9 Conclusions

LSP-maps are specialized geographic maps based on aggregating a number of attribute preferences that characterize the suitability of a geographic location for a specific use. Advantages of LSP-maps can be summarized as follows:

- LSP-maps are general and flexible in the sense that they can express the suitability of the analyzed geographic area for any specific use.
- The method of generating LSP-maps offers a high level of logic versatility originating from the LSP-based soft computing approach. It is easily understandable and consistent with observable properties of human reasoning in the area of evaluation.
- LSP models of suitability generate correct logic results in all points of the attribute space. The accuracy of such models cannot be reduced by unpredictable variations of attribute values. Therefore, the expected reliability of LSP-maps is very good.
- LSP-maps can be dynamically generated from the database of attributes.

- Users of LSP-maps can experiment with various suitability criteria and dynamically investigate effects of changing their parameters.
- As versatile online tools, LSP-maps have potential of becoming an indispensable decision support means in many social, engineering, and business activities.

LSP-maps create various opportunities for future work. The initial efforts should be focused on improving the availability and reliability of input attribute data. There is also space for improving methods for working with incomplete and imprecise attributes. Finally, it is also necessary to develop appropriate software infrastructure that will facilitate the routine creation and experimental use of LSP-maps.

References

Camara G, Palomo G, de Souza R, de Oliveira O (2005) Towards a generalized map algebra: principles and data types, GeoInfo, pp 66–81

Chakhar S, Mousseaua V (2007) An algebra for multicriteria spatial modeling. Comput Environ Urban Syst 31(5):572–596

Delaunay B (1934) Sur la sphère vide. Izvestia Akademii Nauk SSSR. Otdelenie Matematicheskikh i Estestvennykh Nauk 7:793–800

Dujmović JJ (1979) Partial absorption function. J Univ Belgrade EE Dept Ser Math Phys 659:156–163

Dujmović JJ (1987) The LSP method for evaluation and selection of computer and communication equipment. In: Proceedings of MELECON'87, Mediterranean electrotechnical conference and 34th conference on electronics (joint conference), IEEE/RIENA, Rome, Italy, pp 251–254

Dujmović JJ (1991) Preferential neural networks, Chap 7. In: Antognetti P, Milutinović V (eds) Neural networks—concepts, applications, and implementations, vol II. Prentice-Hall advanced reference series. Prentice-Hall, Englewood Cliffs, pp 155–206

Dujmović JJ (2007) Preference logic for system evaluation. IEEE Trans Fuzzy Syst 15(6):1082–1099

Dujmović JJ (2008) Characteristic forms of generalized conjunction/disjunction. In: Proceedings of the IEEE world congress on computational intelligence. IEEE Press, Hong Kong, pp. 1075–1080. ISBN 978-1-4244-1819-0

Dujmović JJ, Nagashima H (2006) LSP method and its use for evaluation of Java IDE's. Int J Approx Reas 41(1):3–22

Frank AU (2005) Map algebra extended with functors for temporal data. In: Perspectives in conceptual modeling. Lecture notes in computer science 3770:194–207

Goovaerts P (1997) Geostatistics for natural resources evaluation. Oxford University Press, New York

Haklay M (2004) Map calculus in GIS: a proposal and demonstration. Int J Geogr Inf Sci (IJGIS) 18(1):107–125

Joerin F, Thériault M, Musy A (2001) Using GIS and outranking multicriteria analysis for land-use suitability assessment. Int J Geogr Inf Sci (IJGIS) 15(2):153–174

Longley P, Goodchild M, Maguire D, Rhind D (2001) Geographic information systems and science. Wiley, NY

Mennis J, Viger R, Tomlin C (2005) Cubic map algebra functions for spatio-temporal analysis. Cartogr Geogr Inf Sci 32(1):17–32

Motro A (1995) Management of uncertainty in database systems. In: Kim W (ed) Modern database systems, the object model, interoperability and beyond. Addison-Wesley, Reading

Rigaux P, Scholl M, Voisard A (2002) Spatial databases with applications to GIS. Morgan Kaufman, Menlo Park

Tomlin C (1990) Geographical information systems and cartographic modelling. Prentice Hall, Englewood Cliffs

Wackernagel H (1995) Multivariate geostatistics—an introduction with applications. Springer, Berlin

Wood LJ, Dragićević S (2007) GIS-based multicriteria evaluation and fuzzy sets to identify priority sites for marine protection. Biodivers Conserv 16:2539–2558