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Adaptive Flood Risk Management Under Climate Change Uncertainty Using Real Options and Optimization

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It is well recognized that adaptive and flexible flood risk strategies are required to account for future uncertainties. Development of such strategies is, however, a challenge. Climate change alone is a significant complication, but, in addition, complexities exist trying to identify the most appropriate set of mitigation measures, or interventions. There are a range of economic and environmental performance measures that require consideration, and the spatial and temporal aspects of evaluating the performance of these is complex. All these elements pose severe difficulties to decisionmakers. This article describes a decision support methodology that has the capability to assess the most appropriate set of interventions to make in a flood system and the opportune time to make these interventions, given the future uncertainties. The flood risk strategies have been explicitly designed to allow for flexible adaptive measures by capturing the concepts of real options and multiobjective optimization to evaluate potential flood risk management opportunities. A state-of-the-art flood risk analysis tool is employed to evaluate the risk associated to each strategy over future points in time and a multiobjective genetic algorithm is utilized to search for the optimal adaptive strategies. The modeling system has been applied to a reach on the Thames Estuary (London, England), and initial results show the inclusion of flexibility is advantageous, while the outputs provide decisionmakers with supplementary knowledge that previously has not been considered.

KEY WORDS: Decision tree analysis; economics; flood risk management; multiobjective optimization; real options

1. INTRODUCTION

Making decisions on long-term flood risk management intervention strategies is complex. Methods are required that are capable of identifying the better performing intervention measures while also taking into account the most effective spatial locations and the most beneficial timing. Given the large portfolio of potential flood risk mitigation measures, identifying the most appropriate long-term strategy is challenging. This problem is further compounded due to the evolving nature of flood risk, in particular with regard to climate and socioeconomic changes. The plausible range of future climate change comprises significant uncertainty, presenting decisionmakers with considerable challenges with regard to long-term planning.

It is widely recognized that the future uncertainties of climate change need to be accounted for within the development of long-term strategies to ensure an economic efficiency.[(1–5)] Traditional approaches do not always lend themselves to adequately account for climate change uncertainty. In the past, strategies were developed without accounting for future uncertainties, including climate change (sea level rise, changes in flood frequency).[(6)] The requirement to account for climate change uncertainty

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has therefore been the subject of significant research[7–9] and methods have been proposed to account for the future uncertainty.[10–12]

Real options analysis is a recognized approach for encouraging appropriate climate change adaptation and mitigation investment decisions.[4,11,13–15] In this article, the concepts of real options and optimization are applied within the context of flood risk management in an estuarine area under climate change uncertainty. This methodology makes use of decision trees and multiobjective optimization to determine flexible and adaptable intervention strategies over a long-term planning horizon.

2. BACKGROUND

2.1. Decision Making Under Severe Uncertainty

The uncertainty in the future climate is significant and its impact on flood risk management decision making is considered to be severe.[16] There are a number of methods that can be applied to aid decision making under severe uncertainty. Wald's[17] maximin or Laplace's principle of indifference[18] are well-known traditional examples. These methods implicitly reflect a particular attitude to uncertainty. Implementation of Laplace's principle is much less conservative compared to that of Wald's maximin, for example. More recently, there has been an increasing trend to develop methods that seek to identify mitigation measures that are described as robust. The concept of robustness, in the context of climate change adaptation, is often not associated with a clear definition; rather, a general concept emerges. The concept generally relates to as having the ability to perform well over a range of future scenarios.

For example, robust decision making (RDM) inverts traditional sensitivity analysis, seeking strategies whose good performance is insensitive to the most significant uncertainties.[19] Hall and Harvey[10] state that a robust option is one that performs well even under future conditions that deviate from our best estimate. Info-gap characterizes uncertainty with nested sets of plausible futures and defines robustness as the range of uncertainty over which a strategy achieves a prescribed level of performance.[20] RDM uses several definitions of robustness, including: (1) trading some optimal performance for less sensitivity to broken assumptions and (2) performing relatively well compared to the alternatives over a wide range of plausible futures.[20]

Many of these authors indicate that there is a distinct choice to be made between robustness and optimization and that robust methods are preferable.[19,21,22]

It is of note, however, that the primary objective of a number of these methods is to maximize robustness;[21] it is thus evident that optimization approaches can be coupled with the general concept of robustness. Extensive research has been undertaken in this regard within the field of robust optimization (RO). RO provides techniques to optimize outcomes while accounting for uncertainties.[23–25] RO is defined by Ben-Tal *et al.*,[24] whereby within, the data are assumed to be "uncertain but bounded," that is, varying in a given uncertainty set, rather than to be stochastic, and the aim is to choose the best solution among those "immunized" against data uncertainty, where Ben-Tal *et al.*[24] refer to "immunized" such that: a candidate solution is "immunized" against uncertainty if it is robust feasible, that is, remains feasible for all realizations of the data from the uncertainty set. It is thus evident that a choice between an optimization and a robustness method is not necessarily required. The objective function of the optimization problem can be defined in terms of robustness criteria that are specified at the outset. This distinction is discussed further by Sniedovich.[26]

Within the analysis described below, the general concept of robustness and optimization is prevalent and hence there are parallels with the RO approach. Note, however, that in a conventional robust optimization approach, which makes use of some fixed, rigid intervention strategy, robustness is achieved by incorporating flexibility within intervention options (i.e., flexibility and the ability to adapt often provides robustness). In the methodology presented here, the robustness (or immunity to uncertainty) is achieved by continuously evaluating the uncertain variable(s) of interest (e.g., sea level rise) and allowing for optional, adaptive/flexible intervention strategies to be implemented/modified in the future, if and when necessary. This can reduce the need for large redundant capacity to be built into the flood defense system.

2.2. Real Options in Flood Risk Management

In flood risk management, a robust strategy is considered to be a strategy that performs well over a range of futures. Performance can be defined using a range of criteria and typically these include strategy costs and benefits. The benefits comprise reduction in risk, where risk can be defined in economic, life loss, and environmental terms. Previous works in this area[2,10,27,28] have sought to develop strategies that are robust to climate change uncertainties. The strategies that have been developed have, however, been fixed over the planning horizon, and although they account for climate change variability, they are based on particular assumptions about future change. The magnitude of future change is, however, subject to severe uncertainty.[29] Rates of change may therefore be faster or slower than the rates assumed and therefore the planned time steps when interventions are required will change. Strategies developed using these approaches may therefore typically require large initial costs and can often result in unnecessary expenditure if a future state occurs that the infrastructure was not tested against.[11]

The core principle of real options analysis is the ability to value flexibility.[30] This principle encourages the identification of opportunities for incorporating flexibility into the decision-making process. Essentially, real options allows a decisionmaker to make changes to an investment decision when new information arises in the future. Opportunities such as *delaying* the investment, *abandoning*, *switching*, *expanding*, *contracting*, or having multiple options interacting together are potential choices for decisionmakers.[31,32] For example, where it is beyond doubt that a flood defense has come to the end of its useful life and requires major refurbishment, there are a range of possible decisions. Assuming a worst-case climate change scenario and constructing a flood defense based on this assumption is likely to be suboptimum as it requires significant up-front expenditure and may well constitute an overdesign should the worst-case scenario not be realized. Constructing a defense that is inherently flexible and capable of future modification is one approach for implementing flexibility within a flood risk system. A flood defense system that is constructed in an innovative way enabling increases in the level of protection to be readily achievable, should there be a requirement, is an example of embedding a real option. The option to raise the level of protection (e.g., raise the crest level) is purchased at the outset. The decision whether to exercise the option is delayed to a future date when more information regarding future climate change impacts, for example, is known. Another example of a real option, in the context of flood risk management, is the purchasing of land adjacent to flood defenses. The option to undertake managed retreat is purchased at the outset. The decision to exercise the option (or not) is then made at a later date when more information is available. A further discussion on these issues is provided by Woodward *et al.*[14]

There may, however, be uncertainty regarding the nature of the mitigation measure. A range of options may exist that could include whether to refurbish a defense, set back a defense, or continue with maintenance activities, the cost of which may rise as the structure approaches the end of its design life. Delaying the decision to refurbish and continue with the maintenance is another example of implementing real-options-based concepts. A delayed decision is preferable in terms of the time value of money and the preference for future investment. Flexibility is maintained and the decision to refurbish or set back is delayed until more information is known. These benefits, however, need to be considered with the potential increase in risk from poorly performing structures and the potential increase in maintenance costs as the structure deteriorates.

There are many methods and tools available to value flexibility and undertake real options analysis. Many are based on financial valuation methods, including the Black-Scholes formula[33,34] and the discrete-time option pricing formula.[35] It is often argued that financial valuation methods such as these are not suitable for valuing real options.[31] Wang and De Neufville[36] explain that real options can be broadly classified into two categories; real options "in" systems and real options "on" systems. Real options "on" systems are real options that focus on the external factors of a system and would benefit most from financial valuation methods. Real options "in" systems, on the other hand, incorporate flexibility into the structural design of the system and valuing this flexibility using financial tools is less suitable. Methods for real options analysis were identified and include partial differential equations,[37] binomial[31] and trinomial[38] decision trees, and stochastic dynamic programming.[39]

In the analysis described below, the use of real options is aligned with real options "in" systems where flexibility is inherently captured within the engineering design of the system. De Neufville *et al.*[40] provide an approach to value flexibility for a real options "in" systems project and the approach adopted in this article follows a similar procedure, evaluating flexibility as the difference between an option with embedded flexibility and an option defined in a more conventional, deterministic way.

In addition to the above, a decision tree approach is also employed enabling real, and other more conventional intervention, options to be incorporated within an intervention strategy, allowing multiple optional intervention paths into the future dependent on the nature and level of climate change. This, in turn, enables more effective adaptation of the analyzed engineering system to climate change.

2.3. Optimization Methods

Formal optimization methods have been applied to flood risk management decision-making problems for many years.[41,42] More recently, evolutionary multiobjective optimization techniques have been developed that have the capability to consider a wide range of multiple objectives simultaneously while searching through a large portfolio of potential decision variables.[43–47] Woodward *et al.*[48] have recently applied the Nondominated Sorting Genetic Algorithm II (NSGAII), an evolutionary multiobjective optimization method,[49] to optimize for short-term flood risk intervention strategies where climate change uncertainty is not a consideration. Multiobjective optimization techniques enable options to be compared over a range of criteria. For example, in flood risk management, relevant criteria include option costs, benefits, life loss, environmental impact (or enhancement), and amenity value. While it is possible to attempt to reduce these criteria to a single monetary measure, the monetization of life, for example, can be particularly controversial. The analysis described here extends upon the work presented by Woodward *et al.*[48] that uses the NSGAII algorithm to aid the development of long-term flood risk strategies where climate change uncertainty is significant. The analysis is performed in terms of benefits and costs using a multiobjective approach that is readily extendable to include additional criteria as required.

3. METHODOLOGY

3.1. Problem

The problem of coastal flood risk management is complex and typically involves a range of performance measures. For the purposes of demonstrating the concepts of the methodology, it is formulated and solved here as a multiobjective objective optimization problem. The two objectives are as follows:

$$f_1(x) = \max(Benefit) \tag{1}$$

$$f_2(x) = \min(Cost) \tag{2}$$

where *Benefit* represents the present value of the reduced flood risk in the analyzed area over a long-term planning horizon (see Equation (5) below) due to the implementation of a specific intervention (or mitigation measure), when compared to the "do nothing" scenario (do nothing is defined as the "walk away" scenario, with no further expenditure). Risk is defined in terms of the expected annual damage (EAD), a measure that is used in standard practice.[50–54] *Cost* represents the present value of the total cost incurred over the same time period due to any interventions implemented and the operation and maintenance costs of the flood defense system (see Equation (11) below).

In order to facilitate the evaluation of flexibility and adaptability, intervention strategies considered are represented as decision trees with multiple paths into the future (see Fig. 1), rather than representing intervention strategies as single paths fixed over the planning horizon. The structure of the adaptable intervention strategy, coded as a decision tree, consists of specific paths at each time step of the planning horizon, where each path or decision node corresponds to a set of intervention measures. Note that these measures are dependent on the uncertain future sea level rise denoting different intervention measures for different cases where the sea level may rise more or less in the future (but not drop down). The intervention measures considered include raising the crest level of the defense (this is constrained based upon the existing defense footprint specification) and enhancing the defense foundation footprint to enable additional crest level raising. In addition, different maintenance regimes of the defenses are also considered.

The intervention measures, coded as decision trees, inherently include flexibility providing opportunities to delay, contract, expand, and abandon investment decisions, depending on how the uncertain future actually unfolds (i.e., how the sea level rises in the case study shown here). Thus, the value of flexibility is explicitly evaluated within the method, thereby incorporating real "in" option analysis. The decision variables within the optimization process not only include the intervention measures but also the threshold values on uncertain climate change variables. This means information on the optimal timing to make an intervention, given the future climate change realization, is provided to decisionmakers.

The decision variables are represented using the following vector:

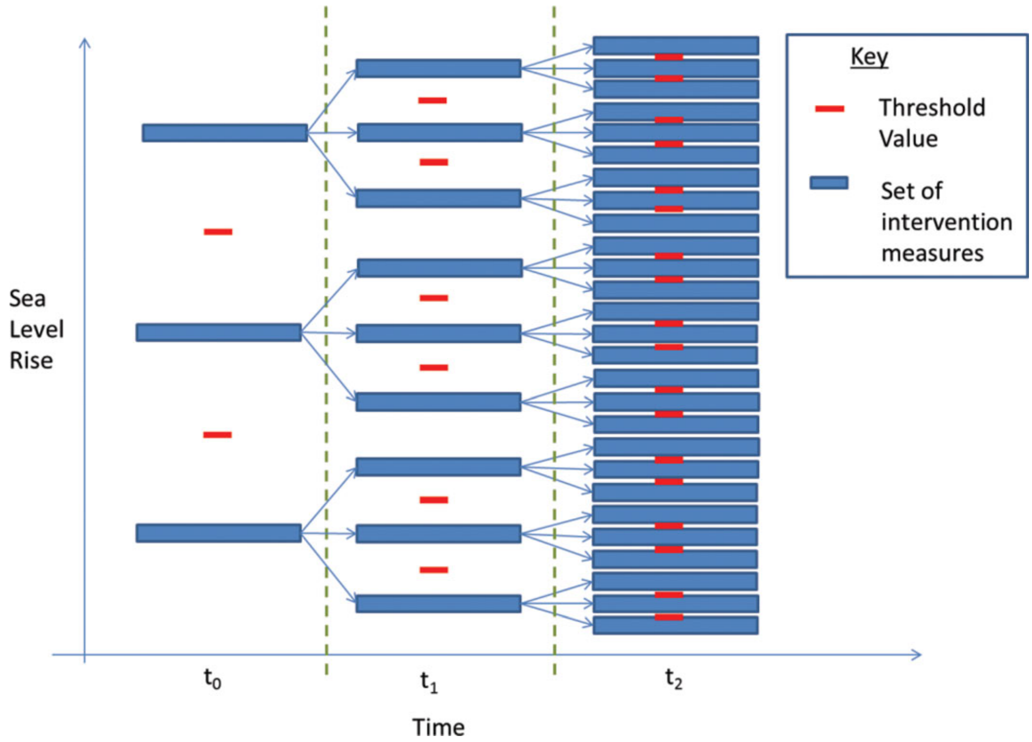

Fig. 1. Intervention strategy represented as decision tree.

$$X = (X_{s,} X_m, T_h) = (x_{s_1}, x_{s_2}, \ldots, x_{s_n}, x_{m_1}, x_{m_2}, \ldots, x_{m_n}, T_{h_1}, \ldots, T_{h_y}) \quad (3)$$

where X_s and $X_{m,}$ are subvectors that represent the specific intervention to apply to each of the defenses, d, in the flood system such that $X_s = (x_{s_1}, x_{s_2}, \ldots, x_{s_n})$ and $X_m = (x_{m_1}, x_{m_2}, \ldots, x_{m_n})$ where n equals the total number of defenses in the flood system, T_h is the threshold value between decision paths, and y is the total number of threshold values. Structural interventions, such as raising the height of a defense, are defined as discrete variables. The decision variable X_m can take the value of four possible maintenance options, including no maintenance, low, medium, and high.

3.2. Climate Change Uncertainty Characterization and Quantification

The decision tree intervention strategies shown in Fig. 1 are evaluated over the three UKCP09 high, medium, and low emission scenarios[55] focusing specifically on sea level rise. The data provided within the three emission scenarios on sea level rise include yearly predicted increases from 1990 to 2100 for the 5th, 50th, and 95th percentiles. For a given emission scenario, the 5th and 95th percentiles are at equidistance from the mean, showing evenly distributed data. A normal distribution was therefore used to represent the uncertainty on sea level rise values for a given emission scenario (see Fig. 2). It was then possible to sample from that distribution to produce a range of future realizations to evaluate the intervention strategies against. For any specific realization, the quantile sampled for the first time step was used for subsequent time steps. This ensured consistency of percentiles at each time step.

Although the three emission scenarios were used, it is important to note that no information on the likelihood of the three scenarios is provided within UKCP09 (see Stainforth[56] for a further discussion on this topic). The approach applied in the case study example was therefore to sample from the three distributions assuming they are equally likely. The methodology is not, however, prescriptive in this regard and consideration of other approaches or weightings is readily achievable.

The uncertainties relating to climate change are accounted for by evaluating each intervention strategy over the full range of future sea level realizations. Given a future realization, the decision path taken is determined according to a threshold value that has

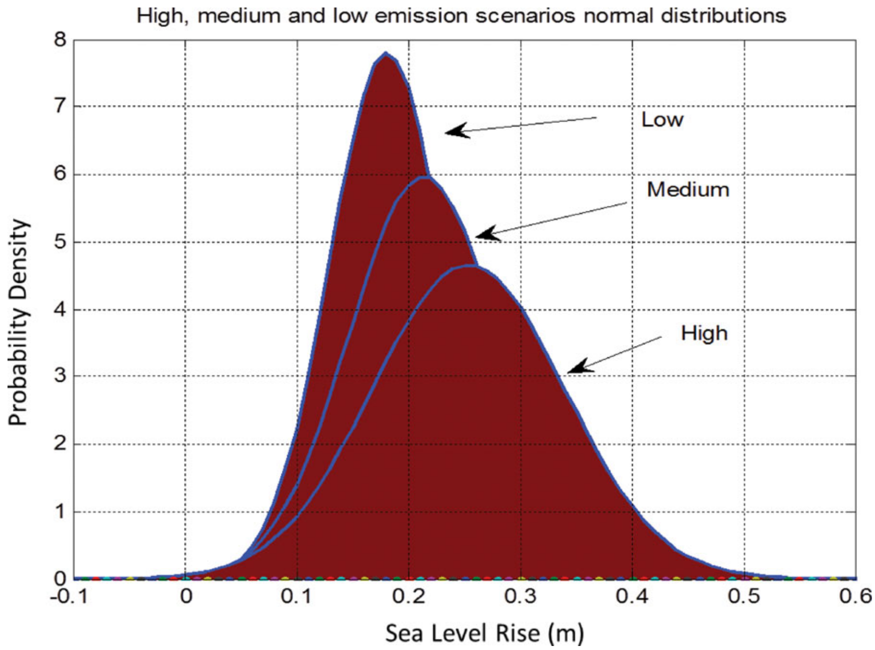

Fig. 2. Normal distributions of sea level rise for each high, medium, and low emission scenario for the year 2030.

been sampled from the normal distributions of sea level rise. At each time step, if the sea level rise of a given realization is greater than the threshold, the higher path is taken, if less, the lower path is taken.

3.3. Flood Risk Assessment

Each adaptable intervention strategy (coded as a decision tree) is evaluated over the range of sampled future scenarios using a risk analysis model and an intervention costing module. The risk analysis model used has been applied to support the development of a long-term flood risk intervention strategy on the Thames Estuary and the Environment Agency's National Flood Risk Assessment.[54]

The model considers a system of flood protection infrastructure protecting the floodplain (Fig. 3). The floodplain is divided into a series of impact zones and further divided into impact cells. The hydraulic loading conditions (e.g., water levels) are represented as continuous random variables acting upon the system of defense sections. The performance of the flood defenses is defined by fragility curves.[57–59] For each hydraulic loading event, it is necessary to consider multiple combinations of defense section failures and overtopped flood defenses. The simulation of flood wave propagation can be computationally time consuming and hence defense system states are sampled using a standard Monte Carlo. The flood wave simulation provides floodplain depths that are then combined with depth damage curves[60] to estimate flood damages. The model evaluates the spatial variation in risk, which is defined as:

$$R = \int \sum_{i=1}^{2^n} P(d_i|l)\, f_L(l)\, g(d_i, l)\, dL, \tag{4}$$

where R is the risk expressed as EAD, in monetary terms (U.K. pounds in the example below), n is the total number of defense sections, l is the hydraulic load at each defense throughout the system, $f_L(l)$ is the probability density function of hydraulic load, d is a specific defense system state, and i is the defense system state index. The function (g) represents the consequences of a single discrete flood event (defined in terms of a specific hydraulic loading level and a defense system state).

The risk analysis model can be used to calculate the present day and future flood risk, accounting for climate change and mitigation measures that are implemented. More specifically, calculation of the flood risk associated with structural and nonstructural interventions, X_s, and routine defense maintenance, X_m, can be incorporated in the model by modifying the fragility curves, defense information, or depth-damage functions. Climate change scenarios are represented by modifying the extreme value

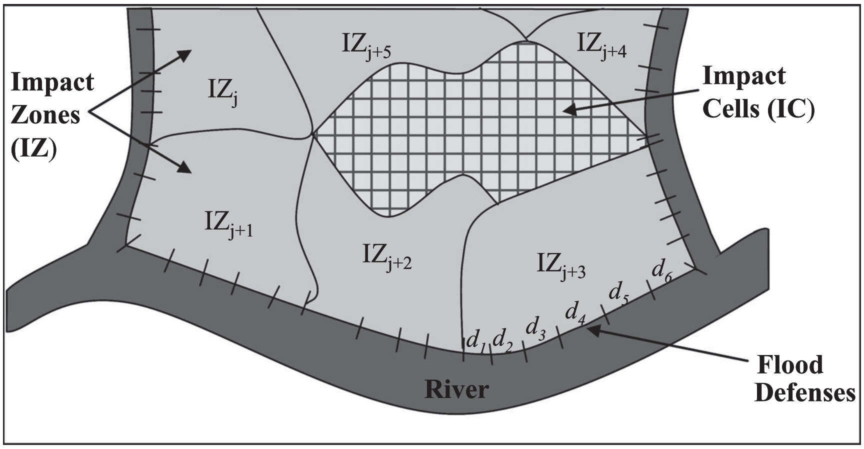

Fig. 3. Conceptual illustration of the modeled system (Gouldby *et al.*, 2008).

distributions of hydraulic loads. While in principle socioeconomic scenarios can be incorporated to a certain degree, by modifying the depth damage scenario, this analysis is not included in the example described below.

For a given climate change realization (e.g., sea level realization), the actual path through the decision tree is determined and the risk analysis model is then used to calculate the associated risk R for that path (see Equation (4).) The risk of a given intervention strategy at any point in time is a function of the intervention measures, the extreme flood events, l, and the performance of the defense infrastructure such that $R = g(X_s, X_m, l,)$. The benefits for that path and given realization can then be obtained as the difference between the "do nothing" option and the path where interventions are applied. The "do nothing" option applies no interventions or defense maintenance over the lifetime of the strategy. The benefits are therefore:

$$Benefit = \sum_{t=1}^{T} \frac{g(X_s, X_m, l, X_p)_t - g(l, X_p)_t}{(1+r)^t}, \qquad (5)$$

where T is the total number of planning horizon time steps considered in an intervention strategy, t represents the time step index, and r is the discount rate.

For each intervention strategy, there is a requirement to run the risk analysis tool for every sea level rise projection to obtain the benefits over a wide range of samples. Depending on the size of the sample, this can become computationally expensive. For this reason, a relationship between the outputs of the risk analysis tool (EAD) and sea level rise has been established for each intervention strategy analyzed to reduce the number of model simulations required. The EAD obtained for each sea level rise sample was found to follow an exponential relationship:

$$y = Ae^{bx}, \qquad (6)$$

where x represents a given sea level rise value, A and b are constants specific to an intervention strategy, and y is the EAD for a given intervention strategy at the sea level rise value x. For each intervention strategy, the flood risk analysis model is run for the maximum and minimum sea level rise values to generate the respective maximum and minimum EAD values. A and b can then be determined using simultaneous equations to produce the exponential relationship for that intervention strategy. It is then possible to determine the EAD values for the remaining sea level rise samples for that intervention strategy using the generated relationship (see, for example, the relationship curve in Fig. 4). The exponential relationship in Fig. 4 gives an R^2 of 0.99 showing the exponential curve fits the data well. The exponential relationship (Equation (6)) was tested for a range of different sea level realizations and different intervention strategies for the case study area below, each time showing consistent results. With this relationship (i.e., surrogate model), it is possible to significantly reduce the overall computational cost as generating a curve for any intervention strategy evaluated requires only two full runs of the risk analysis model.

3.4. Costs

The approach to costing the intervention options developed here identifies costs for 61 different defense classes used within the risk model, which

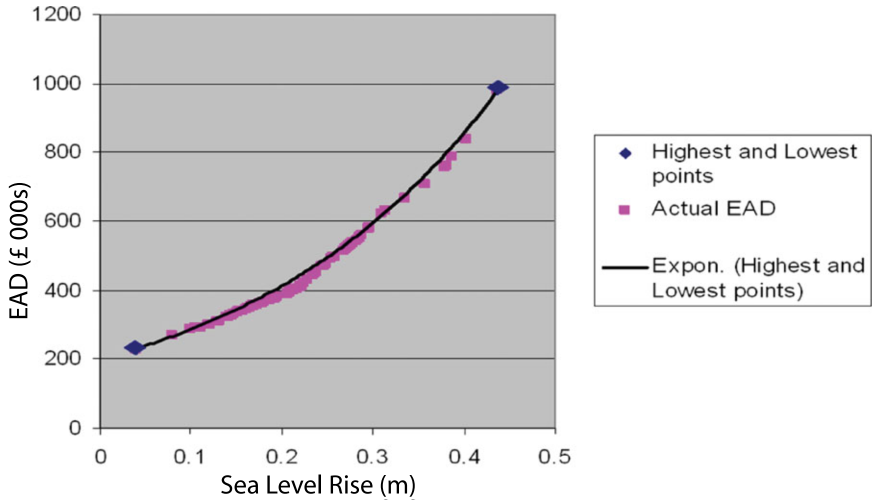

Fig. 4. Exponential relationship between EAD and sea level rise.

were formulated for the National Flood Risk Assessment of England.(61) The basis of the cost model established by Woodward *et al.*(48) extends the cost estimation model given by Phillips.(62) The costs associated with structural interventions, C_s, take into consideration the mobilization (M) and operating costs (O_d), the quantity of work required (Q_j), and the costs of materials (W_j):

$$C_s = M + O_d + \sum_{j=1}^{m} Q_j W_j, \tag{7}$$

where m is the number of maintenance and construction items. The quantity of work required is expressed using the characteristics of the defense such that:

$$Q_j = V_D D_L g(D_x, X_s, G), \tag{8}$$

where V_D are the defense dimensions, D_L is the length of the defense that requires attention, D_x is the severity of the defects, which is a function of the condition grade of the levee, X_s represents the intervention measures being applied, and G is the type of defense being modified. The total overhead and mobilization costs are based on a combination of process published in Langdon(63) and expressed as:

$$M + O_d = \sum_{j=1}^{m} h_j (T_w U_j + M_j) + A, \tag{9}$$

where h_j is the unit number of each mobilization activity, T_w is the number of weeks on site, U_j is the unit cost of each overhead for each mobilization activity, M_j is the mobilization and demobilization cost for each activity, A is the site access costs, and m is again the number of maintenance and construction items.

Maintenance costs, C_m, for four different levels can be evaluated: do nothing, low, medium, and high. The different maintenance levels are reflected within the model by different rates of deterioration associated with the fragility curves.(54) The rates used within this model are obtained from the Environment Agency of England and Wales,(64) also see Hames,(65) with the associated costs obtained from Environment Agency.(66) The total cost, C_t, for a given point in time is simply the maintenance costs plus the structural intervention costs:

$$C_t = C_s + C_m. \tag{10}$$

The total cost of an intervention path sums up the costs at each point in time and then discounts these back to the present day:

$$Cost = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}, \tag{11}$$

where r represents the discount rate, T is the number of time periods, and C_t is the total cost for time period t as defined in Equation (5).

3.5. Implementation of the Optimization Method

The implementation of the optimization algorithm within the context of the methodology proceeds as follows. First, a population of N (500 here) flood risk intervention strategies are generated

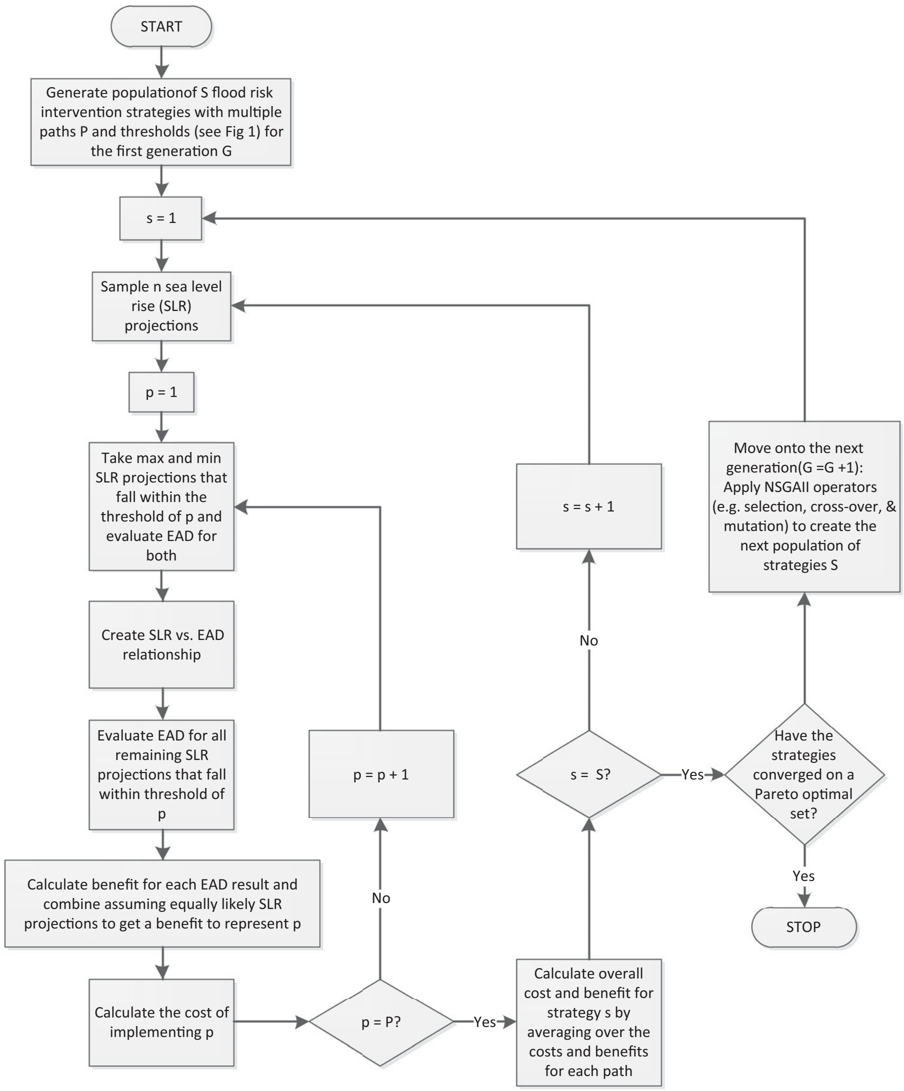

Fig. 5. Flow chart of the methodology.

that follow the structure described in Fig. 1. Each intervention strategy is then evaluated according to its benefits and costs over multiple future scenarios as described above. With each of the *N* initial intervention strategies analyzed according to their objectives (e.g., benefits and costs), the NSGAII operators are applied to create the next generation population of solutions (i.e., strategies). The operators consist of selection, cross-over, and mutation, as shown in Fig. 5. The selection procedure, applied first, determines which strategies will be considered for cross-over and mutation when forming the next generation, with the better performing strategies assigned a higher probability of being selected. To identify the better performing strategies, each strategy is first ranked according to which set of nondominated strategies it is in and, second, according to how close it is to its neighboring strategies in the same rank. A set of solutions is considered to be nondominated (or Pareto optimal) if no other solutions can improve one of the criterion without causing a simultaneous deterioration in another criterion. In the methodology described in this article, binary tournament selection is used whereby two strategies are picked at random and the better performing strategy of the two will survive into the next generation. The process is repeated until a new population of *N* strategies has been created.

Table I. Summary of the NSGA Parameters and Settings Used

Parameter Description	Value
Generations	200
Population size	500
Cross-over type	Bit tournament cross-over
Cross-over rate	0.7
Mutation rate	0.03
Discount rate	Based on the Green Book declining discount rate[71]

Next, the newly selected strategies have the opportunity to undergo cross-over and mutation, to generate new strategies, and prevent convergence on a local optima. These operators are controlled by a probability of occurrence, with cross-over more likely than mutation. The cross-over operator applied in this article is a single-point cross-over where two strategies exchange their setup from a randomly chosen point in their structure. Mutation is then possible, and if it occurs, applies the random replacement procedure. This mutation method randomly modifies a section of the strategy within the bounds of the decision variable range. See Table I for the rates of occurrence used for cross-over and mutation. With the new generation created, the benefit and cost objectives are again evaluated and the process repeated until convergence on a Pareto optimal set has been achieved or a stopping criterion has been met. The overall methodology described in this article is illustrated in a flow chart in Fig. 5.

4. CASE STUDY

4.1. Case Study Description

The methodology has been applied on an area of the Thames Estuary (Fig. 6). The Thames Estuary in London, England is an area that is susceptible to flooding. A large-scale flood event could have a devastating impact as it accommodates over a million residents and workers, 500,000 homes, and 40,000 nonresidential properties.[61,67–69] The threat of flooding on the Thames Estuary occurs from a number of different sources, including high sea levels and surges propagating from the North Sea into the Estuary and extreme fluvial flows along the Thames and its tributaries.[70] Protection against flooding is provided by a range of fixed defenses and actively operated barriers and flood gates. The majority of the defenses were designed to protect against a 1-in-1,000 year flood; however, at the present day these flood defenses are gradually deteriorating. In the longer term, with the potential impacts of climate change, the need to consider a range of intervention measures is evident. It is, however, recognized in the planning for the future of the Thames Estuary that the decisions made today can impact the ability to adapt in the future. The Thames Estuary is therefore a suitable case study to investigate the use of the real options concepts and optimization methods described in this article for flood risk management.

For reasons of computational practicality, this study focuses on a specific reach, Thamesmead, within the Estuary (Fig. 6). It is important to note that some data have been somewhat modified and hence the results presented here do not reflect the true risk within Thamesmead. This area contains 79 defenses, which have been classified into five groups according to defense characteristics and location. The defense characteristics that influence the groupings of the defense are the defense type and condition grade. The defense types include brick and masonry and sheet pile vertical walls, and rip-rap and rigid embankments.

The case study looks at two different situations (cases 1 and 2). First, the optimization model is applied in a deterministic manner whereby only one future climate change realization is considered, the 50th quartile of the high UKCP09 emission scenario. For this case where it is assumed that the future is certain, there is no requirement to build in flexibility and thus use a decision tree structure. The strategy is instead defined as a single fixed path over the planning horizon. The second case assumes the future is uncertain and therefore considers multiple future realizations, adopting the decision tree structure for the intervention options to enable flexibility in long-term planning. Two differing future paths are considered for this second case to demonstrate the real options decision tree approach where each future path represents a possible investment route into the future. A comparison of the two cases is also undertaken.

In both cases, the intervention strategies consider a planning horizon of 100 years with intervention measures considered at every 50-year time step. The decision variables that are considered within the intervention strategies include raising the crest level of defenses, increasing the capacity of the defenses for future expansion, and the level of maintenance

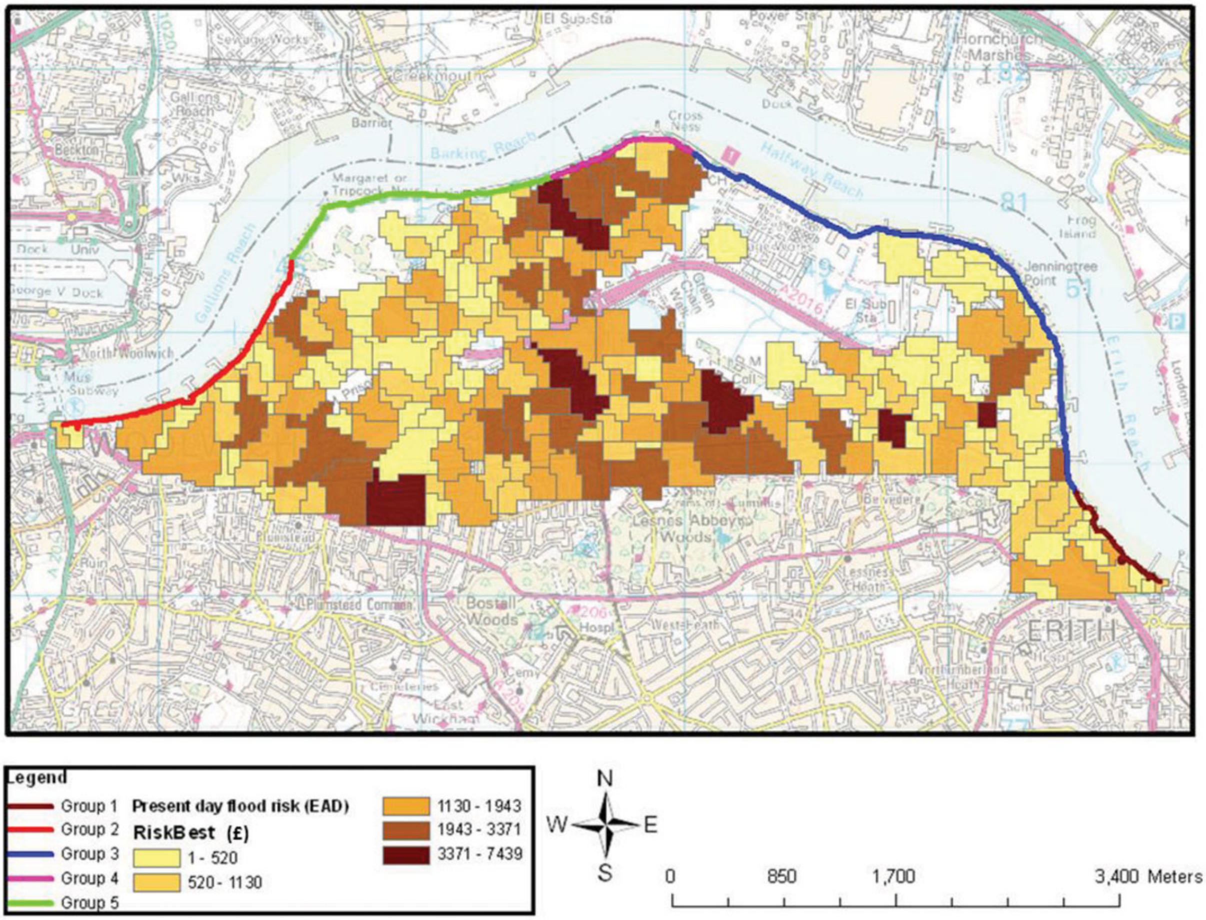

Fig. 6. The present day flood risk (obtained using the flood risk assessment method explained in Section 3.3) to the flood area of interest on the Thames Estuary with the five groups of defenses protecting the floodplain.

applied. The NSGAII parameters and settings used for all optimization runs are summarized in Table I.

4.2. Results and Discussion

4.2.1. Case 1

Fig. 7 displays the optimal Pareto front obtained in case 1 evaluated against one future realization, showing the tradeoff between flood risk reduction and costs. A range of intervention strategies on the Pareto front have been highlighted, including the strategy with the highest net present value (NPV; triangle) and the highest benefit-cost ratio (BCR; square) for illustrative purposes. NPV is the present value of the net benefit (difference between benefit and cost).

Using the respective positioning of these strategies on the Pareto front, decisionmakers can make a well-informed decision, comparing the different strategies available to select the most appropriate. A solution cannot be improved with respect to one objective without causing a negative effect on the other objective. For example, improving the benefit will result in an increase in the cost. Decisions can also be determined according to specific target levels that must be met for each criterion. For example, a specific flood risk reduction level that must be reached or if there is a constraint in the total expenditure allowed.

Table II displays a summary of the five optimal strategies from the Pareto front that have been highlighted. Comparing strategies C and D, it can be seen that for a minimal increase in cost, the benefits in terms of flood risk reduction can be significantly

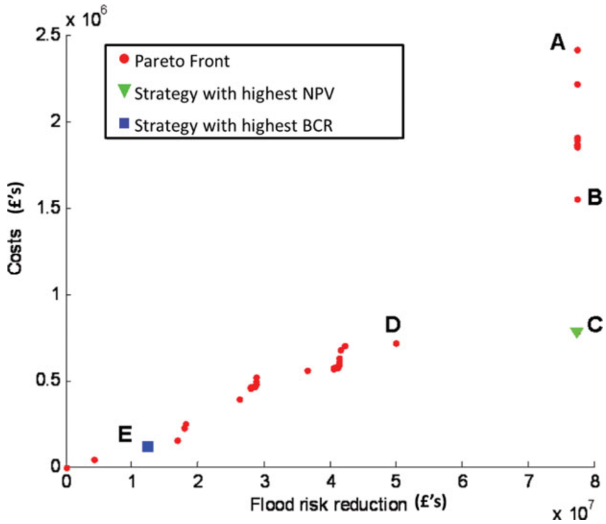

Fig. 7. Pareto front obtained using deterministic optimization approach.

improved, favoring strategy C. Similarly, comparing strategy B and C, the increase in benefits for strategy B does not outweigh the considerable increase in costs.

The suggested intervention measures for these five strategies vary (see Table II). Strategy E, for example, applies the minimum number of intervention options, only applying a low-maintenance regime, and achieves the highest BCR. For an increase in cost and a large increase in flood risk reduction, strategy D applies a medium level of maintenance instead of a low level. To achieve a further increase in flood risk reduction, structural interventions are required.

Strategies A, B, and C comprise either a low or medium maintenance over the 100 years as well as a height increase to at least one group of defenses in at least one of the time steps. In all three solutions, the defenses in group 1 are increased by 1.33 m. Group 1 defenses protect a highly developed area in a vulnerable location to storm surges and increasing the height of these defenses enables a significant amount of the risk to be reduced.

4.2.2. *Case 2*

Fig. 8 displays results from case 2, where the Pareto front of the 200th generation was optimized for flexible long-term strategies, which inherently capture the real options concepts. A total of 1,000 sea level rise samples were used to evaluate each intervention strategy on the Pareto front. Four intervention strategies on the Pareto front have been identified, strategies A to D, including the strategy with the highest NPV (triangular point) and the highest BCR (square point). Table III displays the benefits, costs, NPV, and BCR for these strategies while Fig. 9

Table II. Summary of the Benefits, Costs, NPV, BCR, and Intervention Measures of Select Strategies from the Pareto Front Highlighted in Fig. 7

Strategy	Benefit (£M)	Cost (£M)	NPV (£M)	BCR	Intervention Measures
A	77.31	2.41	74.89	32.08	Time step 1 Raise G1 by 1.33 m, G2 by 1.00 m, and G4 by 0.33 m Time step 2 Raise G3 by 0.66 m Medium maintenance to G3 and G4
B	77.29	1.55	75.74	49.86	Time step 1 Raise G1 by 1.33 m, apply medium maintenance to G3 Time step 2 Apply medium maintenance to G3
C	77.28	0.79	76.49	97.82	Time step 1 Low maintenance to G1, G3, and G4 Time step 2 Raise G1 by 1.33 m Low maintenance to G3
D	49.87	0.72	49.15	69.26	Time step 1 Medium maintenance to G1, G3, and G4 Time step 2 Medium maintenance to G3
E	12.53	0.11	12.42	113.91	Time step 1 Low maintenance to G1, G3, and G4

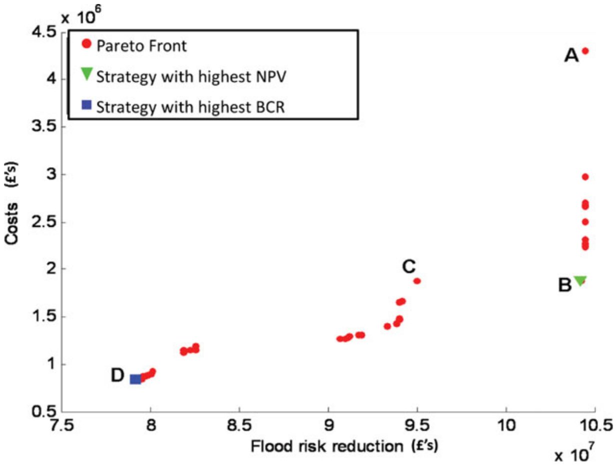

Fig. 8. Pareto front obtained using real-options-based optimization.

Table III. The Benefits, Costs, NPV, and BCR of the Solutions Highlighted in Fig. 8

Strategy	Benefit £M	Cost £M	NPV £M	BCR
A	104.45	4.31	100.14	24.23
B	104.22	1.88	102.34	55.44
C	94.97	1.87	93.10	50.79
D	79.23	0.84	78.39	94.32

displays the structure of each of the four solutions and the intervention measures for each path.

Strategy B obtains the highest NPV. This strategy comprises the incorporation of refined foundations to three groups of defenses at the first time step, to enable further elevation increase, as well as raising two of these groups. At the next time step of strategy B, the bottom path represents a "do nothing" option, which is the chosen path for sea level realizations with a rise less than 0.37 m. In this case, if the sea level rise increase does not go beyond this threshold no additional investment needs to be spent on interventions. For the sea level realizations, that have a sea level greater than 0.37 m, the top intervention path is taken where the defense's crest levels will be raised. Sixty-one percent of the 1,000 sea level rise samples were directed to the top path while only 29% took the bottom. For strategies C and D, it is also recommended that if the sea level rises above 0.37 m it is optimal to take the top path, otherwise take the bottom.

Strategy A on the other hand comprises taking the top path if the sea level rise increase goes beyond 0.52 m, otherwise take the bottom path. Strategy B achieves a very similar benefit compared to strategy A but for a significantly lower cost, which improves the overall NPV. The difference in cost can be attributed to the way the flexibility is used. Strategy A here does not purchase the "insurance policy" for the second time step (i.e., does not extend the defense's footprint at the first time step in order to have the opportunity at a later date to increase the height). Instead strategy A delays any decision to widen or raise the defense. For strategy A, if the sea level rise is beyond the threshold, a greater capacity for crest level raising therefore needs to be introduced. This requires additional costs. Although the option is flexible in that a decision is delayed until more is known about the future impacts of climate change, the costs in the way this flexibility is used are less favorable. In particular, it is important to note that the decision to delay, while affording flexibility, incurs an increase in risk (hence less benefit), in the near term. Strategies B and C instead purchase this "insurance policy" to enable flexibility to be inherently built into the defenses. Strategy B is then able to achieve similar benefits to strategy A but for a reduction in costs of 56% and thus shows strategy B to be more favorable.

In this case study, strategy A applies real "on" options using a delay in the investment. Flexibility is not built into the design of the defenses as the defense infrastructure needs to be modified in the second time step if the top path is taken. Strategy C applies real "in" options by building flexibility into the design of the system. In the second time step, the defense can be easily adapted to account for an increase in sea level rise.

This inclusion of flexibility, real "on" options, can increase the cost of the investment compared to strategies without flexibility and also incur higher risks in the near term. In this example, even with the increase in cost, the incorporation of flexibility can still improve the overall investment decision; this can be seen through the comparison of the case 1 and 2 results.

4.2.3. Comparison of Cases 1 and 2

In order to compare the adaptable strategies (i.e., strategies obtained assuming an uncertain future) with the deterministic strategies (i.e., strategies obtained assuming a certain future), the Pareto fronts obtained using the two approaches have been reevaluated with the same set of 1,000 future sea level rise samples. This enables the comparison of

Strategy A

G1	G2	G3	G4	G5
1.33	0.00	0.00	1.33	0.33
0.5	1.0	0.5	3.0	0.5
0	L	0	L	L

G1	G2	G3	G4	G5
1.33	0.00	0.33	0.46	0.36
0.5	2.5	0.5	2.5	3.0
M	0	0	0	M

Threshold Value:
0.52m

G1	G2	G3	G4	G5
1.00	0.36	0.00	0.00	0.30
0.0	0.0	0.0	0.0	0.0
0	L	0	0	L

2010 T_1 2060 T_2 2100

Strategy B

G1	G2	G3	G4	G5
1.33	0.00	0.00	0.00	0.33
1.5	0.0	0.0	3.0	0.5
0	0	0	0	0

G1	G2	G3	G4	G5
1.33	0.00	0.00	0.00	0.46
0.5	0.0	0.0	0.0	0.0
M	0	0	0	0

Threshold Value:
0.37m

G1	G2	G3	G4	G5
0.00	0.00	0.00	0.00	0.00
0.0	0.0	0.0	0.0	0.0
0	0	0	0	0

2010 T_1 2060 T_2 2100

Strategy C

G1	G2	G3	G4	G5
0.00	0.00	0.00	0.33	0.00
1.0	0.5	0.0	3.0	0.5
0	0	L	0	0

G1	G2	G3	G4	G5
1.33	0.00	0.00	0.00	0.46
0.0	0.0	0.0	0.0	0.0
M	0	0	0	0

Threshold Value:
0.37m

G1	G2	G3	G4	G5
0.46	0.00	0.00	0.00	0.00
0.0	0.0	0.0	0.0	0.0
0	0	0	0	0

2010 T_1 2060 T_2 2100

Strategy D

G1	G2	G3	G4	G5
0.00	0.00	0.00	0.00	0.00
0.0	0.0	0.0	3.0	0.5
0	0	0	0	0

G1	G2	G3	G4	G5
1.33	0.00	0.00	0.00	0.46
0.5	0.0	0.0	0.0	0.0
L	0	0	0	0

Threshold Value:
0.37m

G1	G2	G3	G4	G5
0.30	0.00	0.00	0.00	0.00
0.0	0.0	0.0	0.0	0.0
0	0	0	0	0

Fig. 9. Summary of the intervention strategies identified in Fig. 8. Each strategy is a decision tree with two optional paths at the second time step (T_2) with the percentage of samples evaluated at each path undertaken. The first row of each block represents the group (G) where the interventions are being implemented, the second row represents height increases in meters, the third row represents width increases in meters, and the final row represents the defense maintenance (0 = no maintenance, L = low, M = medium, H = high).

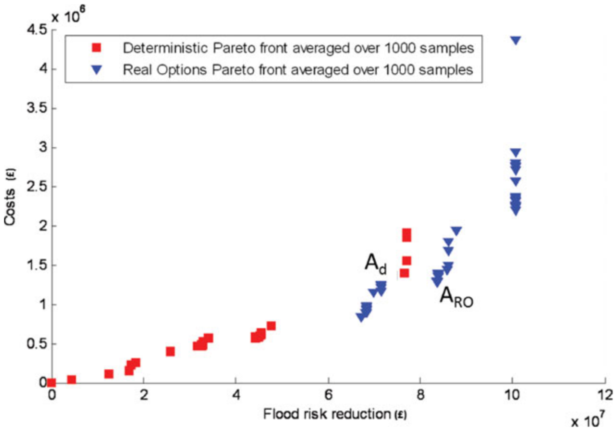

Fig. 10. The Pareto front of the real options optimization and deterministic optimization.

Table IV. A Comparison of Two Solutions from the Real Options Pareto Front and Deterministic Pareto Front When Evaluated Over the Same 1,000 Climate Change Scenarios as Highlighted in Fig. 10

Strategy	Benefit £M	Cost £M	NPV £M	BCR
A_d	47.76	0.72	47.04	66.33
A_{RO}	67.20	0.83	66.37	80.96
% difference	28.93	13.25	29.12	18.07
B_d	76.58	1.38	75.20	55.49
B_{RO}	83.81	1.40	82.41	59.86
% difference	8.63	1.43	8.75	7.97

the performance of the two sets of solutions in a like situation. Fig. 10 displays the two reevaluated Pareto fronts. From this figure, it can be seen that the inclusion of flexibility within the intervention strategies has increased the overall cost of the solutions when there is an uncertain future. This inclusion of flexibility does, however, also provide the opportunity to significantly increase the benefits in terms of flood risk reduction, resulting in a considerable improvement to the overall investment. For example, the decision-tree-based optimization overall has been able to obtain solutions with significantly higher benefits than the deterministic approach. This is partly due to the additional optional paths in the decision tree solutions. Each path can be optimized to a smaller range of climate change samples and therefore provide better flood protection. Additionally, the deterministic solutions were optimized according to one climate change realization and therefore when analyzing the solutions over a range of samples, it is likely that these solutions will not fair so well under different samples and thus bring in less benefits.

For example, strategies A_d and A_{RO} have similar costs (differ only by 0.7%) but the flexible strategy A_{RO} returns a larger benefit by 8% and again improves the NPV, this time by 9% (see Table IV). Strategy A_d only raises and widens the defenses in group 1 by 1 m. A_{RO} is able to widen the base of the defenses in groups 1 and 4 in the first time step, then in the second time step decides on the height of the crest level increase according to the climate change realization. If the sea level increases beyond 0.56 m, it is suggested the defenses are raised by 1 m in group 1 and apply maintenance to group 4 whereas if it does not go beyond this threshold, a raise of 0.66 m to group 1 is suggested. Having the flexibility within the strategy enables a more effective investment to be planned.

From this example, it can be seen that with similar costs, the adaptable strategies (coded as decision trees) that make use of the real options concept will return higher benefits and thus dominate (in the Pareto sense) the deterministic, rigid strategies. This is because the decision tree solutions have been designed to account for the future uncertainties of climate change by developing alternative, customized strategies appropriate for specific realizations of climate change, thus covering, in a flexible manner, a large range of possible future realizations. In addition to this, the concept of real options, which effectively acts as an insurance policy, is ensuring that the options available to the decisionmaker are kept open in the future (at a cost), i.e., that certain intervention options can be implemented later on, if, when, and in the quantity required. The deterministic solutions on the other hand were developed based on a single forecasted future realization only and without allowing for any flexibility in the intervention strategy. Therefore, in the face of uncertainty where many different scenarios could potentially occur, the deterministic solutions may not be sufficient. These are therefore not as favorable and have been shown to be dominated by solutions that account for the future uncertainties of climate change.

5. CONCLUSIONS

This article describes a new methodology to support decision making in long-term flood risk management. An existing flood risk assessment model has been coupled with a costing model and an NSGAII multiobjective optimization algorithm. The concepts of real options and adaptive engineering design with intervention strategies represented using decision trees specified over the predefined planning horizon have then been applied to create the new methodology. The resulting system trials different flexible intervention measures, using the intelligent option searching characteristics of the NSGAII; it then evaluates the costs associated with the interventions and their benefits, in terms of flood risk reduction taking account of future climate change uncertainty. This process is iterated until a Pareto front, or "trade-off" curve, is formed producing optimal decision tree strategies for flood risk management.

The decision trees display the most appropriate intervention measures at various planning horizon time steps depending on how the future unfolds. Threshold values are optimized to determine, given a future projection, which intervention route is best to follow. The use of real options analysis enables the flexibility within the decision trees to be valued and thus account for the future uncertainties of climate change.

The use of evolutionary multiobjective optimization algorithms has the potential to provide a greater range of information to decisionmakers. The system is capable of outputting a set of trade-off solutions, which present a range of potential flood risk mitigation intervention strategies. Each strategy is optimal according to given criteria (costs, benefits) and presents information describing the most appropriate intervention measures to implement, when and where. The application of the new methodology in area of the Thames Estuary demonstrates the benefits that real options optimization can bring to flood risk management decision making.

Future work will include applying the methodology developed and presented here to even more complex real-life case studies with wider range of intervention measures considered and more detailed decision tree structures considered. Future work will also consider transferring some of the concepts shown here to other water engineering systems (e.g., urban water infrastructure systems).

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