Variable Cosmological Constant
as a Planck Scale Effect

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Abstract. We construct a semiclassical Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model assuming a running cosmological constant (CC). It turns out that the CC becomes variable at arbitrarily low energies due to the remnant quantum effects of the heaviest particles, e.g. the Planck scale physics. These effects are universal in the sense that they lead to a low-energy structure common to a large class of high-energy theories. Remarkably, the uncertainty concerning the unknown high-energy dynamics is accumulated into a single parameter ν, such that the model has an essential predictive power. Future Type Ia supernovae experiments (like SNAP) can verify whether this framework is correct. For the flat FLRW case and a moderate value $\nu \sim 10^{-2}$, we predict an increase of $10 - 20\%$ in the value of Ω_Λ at redshifts $z = 1 - 1.5$ perfectly reachable by SNAP.

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Introduction

The number and versatility of publications concerning the Cosmological Constant (CC) problem, show that it is a fascinating interface between Cosmology and Quantum Field Theory (QFT). One can identify three distinct aspects of this problem: I) The famous "old" CC problem (see [1]) is why the induced and vacuum counterparts of the CC cancel each other with such a huge precision; II) The coincidence problem (see the reviews [2]) is why the observed CC in the present-day Universe [3] is so close to the matter density ρ; III) The dynamics responsible for the CC problem: is the CC some form of dark energy associated to a special new entity? If future astronomical observations (like the SNAP project [4]) would discover that the CC depends on the redshift parameter z, one could wonder whether this variation can be achieved only through the mysterious quintessence scalar field searched by everyone. The idea of a scalar field that may adjust itself such that the cosmological constant is zero or very small, has been entertained many times in the literature and since long ago [5, 6]. But in practice one ends up with all sorts of (more or less obvious) fine-tunings. As emphasized in [7, 8, 9], the quintessence proposal for problem III does not help much in solving problems I and II. In our opinion, before introducing an *ad hoc* field we had better check whether the variation of the CC can be attributed to a simpler, more economical and truly robust QFT concept, like e.g. the renormalization group (RG).

The relation between the RG and the CC problem had been in air for a long time [10, 11], but only recently it has been settled into the phenomenological framework [8, 9, 12, 13]. A consistent formulation of the approach within the context of particle physics in curved space-time has been presented in [9], where the relevant RG scale μ has been identified with the energy of the gravitational quanta; that means, in the cosmological arena, with the Hubble parameter: $\mu = H$. Other possibilities have been considered in the literature and in different contexts [3]. At first sight, the tiny present-day value $H_0 \sim 1.5 \times 10^{-42}\, GeV$ makes the RG running of the CC senseless, because all the massive fields should decouple. Of course, this is true if we apply the "sharp cut-off" scheme of decoupling [8]. However, in [12] a pertinent observation concerning the decoupling has been made. In fact, the decoupling is not sharp, and the quantum effects of a particle with mass m at the energy $\mu \ll m$, even if suppressed by the factor $(\mu/m)^2$, can be important in the CC context. Recent investigations of the decoupling in the gravitational theory [14] have confirmed explicitly this decoupling behavior for the higher-derivative sector of the vacuum action. However, the running of the CC and the inverse Newton constant is not visible within the perturbative approach of [14], while alternative covariant methods of calculations are incompatible with the mass-dependent renormalization scheme and their development may require a long time. In this situation it is reasonable to apply the phenomenological approach. In the present paper we suppose that the decoupling of the CC occurs in the same way as in those sectors of the vacuum action, where we are able to obtain this decoupling explicitly [14]. As we are going to show, this approach may shed light to the coincidence problem II and leads to very definite predictions for

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[3]In Ref.[8] the alternative possibility associating μ with $\rho^{1/4}$ is put forward, and in [12] it is amply exploited. In Ref.[13] the RG scale μ was identified with the inverse of the age of the universe at any given cosmological time, i.e. $\mu \sim 1/t$. This is essentially equivalent to our choice [9], because $H \sim 1/t$ in the FLRW cosmological setting.

the variable CC, which can be tested by the rich program of astronomical observations scheduled for the next few years.

Renormalization Group and Planck Scale Physics

Consider a free field of spin J, mass M_J and multiplicities $(n_c, \, n_J)$ in an external gravitational field – e.g. $(n_c, n_{1/2}) = (3, 2)$ for quarks, $(1, 2)$ for leptons and $(n_c, n_{0,1}) = (1, 1)$ for scalar and vector fields. At high energy scale, the corresponding contribution to the β-function $d\Lambda/d\ln\mu$ for the CC is [9]

$$\beta_\Lambda(\mu \gg M_J) \;=\; \frac{(-1)^{2J}}{(4\pi)^2} \, (J + 1/2) \, n_c \, n_J \; M_J^4 \,. \tag{1}$$

At low energies $(\mu \ll M_J)$ this contribution is suppressed due to the decoupling. At an arbitrary scale μ (whether greater or smaller than M_J), the contribution of a particle with mass M_J should be multiplied by a form factor $F(\mu/M_J)$. At high energies $F(\mu \gg M_J) \simeq 1$ because there must be correspondence between the minimal subtraction scheme and the physical mass-dependent schemes of renormalization at high energies. In the low-energy regime $\mu \ll M_J$ one can expand the function F into powers of μ/M_J:

$$F\left(\frac{\mu}{M_J}\right) \;=\; \sum_{n=1}^{\infty} k_n \left(\frac{\mu}{M_J}\right)^{2n} \,. \tag{2}$$

Two relevant observations are in order. First, the term $n = 0$ must be absent, because it would lead to the non-decoupling of M_J, with untenable phenomenological implications on the CC value. Indeed, for those terms in the vacuum action where the derivation of the function $F(\mu/M_J)$ is possible [14], the $n = 0$ terms are really absent. Second, due to the covariance the number of metric derivatives (resulting into powers of H) must be even, and so there are no terms with odd powers in the expansion (2) – remember we identify μ with the Hubble parameter. No other restrictions for the coefficients k_n can be seen. Indeed, in the $H \ll M_J$ regime the most relevant coefficient is k_1. In the rest of this article we develop a cosmological model based on the hypothesis that this coefficient is different from zero. This phenomenological input does not contradict any known principle or law of physics and at the present state of knowledge its validity can be checked only by comparison with the experimental data.

As far as we suppose the usual form of decoupling for the CC, all the contributions in (1) are suppressed by the factor of $(\mu/M_J)^2$ and we arrive at the expression

$$\beta_\Lambda(\mu \ll M_J) \;\simeq\; k_1 \, \frac{(-1)^{2J}}{(4\pi)^2} \, (J + 1/2) \, n_c \, n_J \, M_J^2 \, \mu^2 \,. \tag{3}$$

At the very low (from the particle physics point of view) energies $\mu = H_0 \sim 1.5 \times 10^{-42} \, GeV$, the relation $(\mu/M_J)^2 \ll 1$ is satisfied for all massive particles: starting from the lightest neutrino, whose presumed mass is $m_\nu \approx 10^{30} \, H_0$, up to the unknown heaviest particle $M_+ \lesssim M_P$. Then, according to (3), the total β-function for the CC in the present-day Universe is, in very good

approximation, dominated by the heaviest masses:

$$\beta_\Lambda^t = \sum_{M_J=m_\nu}^{M_+} \beta_\Lambda(M_J) \simeq \frac{1}{(4\pi)^2} \, \sigma \, M^2 \, \mu^2 \,. \tag{4}$$

Here we have introduced the following parameters: M is a mass parameter which represents the main feature of the total β-function for the CC at the present cosmic scale, and $\sigma = \pm 1$ indicates the sign of the CC β-function, depending on whether the fermions ($\sigma = -1$) or bosons ($\sigma = +1$) dominate at the highest energies. The quadratic dependence on the masses M_J makes β_Λ^t highly sensitive to the particle spectrum near the Planck scale while the spectrum at lower energies has no impact whatsoever on β_Λ^t.

Having no experimental data about the highest energies, the numerical choice of σM^2 is model-dependent. For example, the fermion and boson contributions in (4) might cancel due to supersymmetry (SUSY) and the total β-function becomes non-zero at lower energies due to SUSY breaking. In this case, the value of M^2 depends on the scale of this breaking, and the sign σ depends on the way SUSY is broken. In particular, the SUSY breaking near the Fermi scale leads to a negligible β_Λ^t, while the SUSY breaking at a GUT scale (particularly at a scale near the Planck mass) provides a significant β_Λ^t.

Another option is to suppose some kind of string transition into QFT at the Planck scale. Then the heaviest particles would have the masses comparable to the Planck mass M_P and represent the remnants, e.g., of the massive modes of a superstring. Of course, this does not contradict SUSY at the lower energies, when these string WIMPzillas decouple from the matter sector. But, according to Eq. (4), they never decouple completely from the RG in the CC case. Below, we shall work with this option and take, for simplicity, $M^2 = M_P^2$. Let us clarify that this choice does not necessary mean that the relevant high energy particles have the Planck mass. The mass of each particle may be indeed smaller than M_P, and the equality, or even the effective value $M \gtrsim M_P$, can be achieved due to the multiplicities of these particles. With these considerations in mind, our very first observation is that the natural value of the β-function (4) at the present time is

$$\left| \beta_\Lambda^t \right| = \frac{1}{(4\pi)^2} \, M^2 \cdot \mu^2 = \frac{c}{(4\pi)^2} \, M_P^2 \cdot H_0^2 \sim 10^{-47} \, GeV^4 \,, \tag{5}$$

where c is some coefficient. For $c = \mathcal{O}(1-10)$ the β_Λ function is very much close to the experimental data on the CC [3]. This is highly remarkable, because two vastly different and (in principle) totally unrelated scales are involved to realize this "coincidence": H_0 (the value of μ at present) and M_P, being these scales separated by more than 60 orders of magnitude! In this framework the energy scale of the CC is of the order of the geometrical mean of these two widely different ones, and only near M_P all scales become of the same order. Moreover at low energy the running of the CC proceeds smoothly and in the right ballpark required by the natural solution of problem II. In fact, this is not a complete solution of this problem. Of course Eq. (4) holds at any cosmic scale, but the Friedmann equation leads to the relation $H \sim \sqrt{\Lambda}/M_P$ only at lower energies, while at higher energies the matter or radiation density dominates. Anyway, the coincidence between the supposed RG decoupling (4) and the Friedmann equation for the modern, CC-dominated, Universe looks rather intriguing and is worth exploring.

A cosmological FLRW model with running Λ

Consider the implications for FLRW cosmological models coupled to Eq. (4). The first step is to derive the CC dependence from the redshift parameter z, defined as $1 + z = a_0/a$, where a_0 is the present-day scale factor. Using the identification of the RG scale μ with H, we reach the equation

$$\frac{d\Lambda}{dz} = \frac{1}{H}\frac{dH}{dz}\beta_\Lambda^t = \frac{\sigma M^2}{(4\pi)^2}H\frac{dH}{dz}\,. \tag{6}$$

In order to construct the cosmological model, we shall use, along with Eq. (6), the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) - \frac{k}{a^2}\,, \tag{7}$$

where ρ is the matter/CDM/radiation density and the last curvature-dependent term can be presented as $-k/a^2 = H_0^2\,\Omega_K^0(1+z)^2$, where Ω_K^0 is the spatial curvature parameter at present ($z = 0$), and can be written in terms of the usual cosmological parameters for matter and CC at the present time: $\Omega_K^0 = 1 - \Omega_M^0 - \Omega_\Lambda^0$. Also, the energy conservation law provides the third necessary equation

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0\,, \tag{8}$$

where p is the matter/radiation pressure. One could wonder whether the non-local effects behind the renormalization group are compatible with the standard energy conservation equation (8). Let us remember, however, that the covariant form of the conservation law $< \nabla_\mu T_\nu^\mu >= 0$ just reflects the covariance of the effective action and therefore does not depend on the non-localities which are always present in the quantum corrections. Furthermore, in a situation where the energy scale associated to the metric derivatives is very small, the leading effect of the CC scale dependence may be, according to our model, presented in a compact form due to the renormalization group. Then the proper form of the conservation law is the one we use here.

As we shall consider both MD (matter dominated) and RD (radiation dominated) regimes, it is useful to solve the equations (6), (7), (8) using an arbitrary equation of state $p = \alpha\rho$, with $\alpha = 0$ for MD and $\alpha = 1/3$ for RD. The time derivative in (8) can be easily traded for a derivative in z via $d/dt = -H(1+z)\,d/dz$. Hence we arrive at a coupled system of ordinary differential equations in the z variable. The solution for the matter-radiation energy density and CC is completely analytical. It reads a follows:

$$\rho(z;\nu) = \left(\rho_0 + \frac{\kappa}{\zeta - 2}\rho_c^0\right)(1+z)^\zeta - \frac{\kappa}{\zeta - 2}\rho_c^0(1+z)^2 \tag{9}$$

and

$$\Lambda(z;\nu) = \Lambda_0 + \frac{3\,\nu}{8\,\pi}M_P^2\left(H^2(z;\nu) - H_0^2\right) = \Lambda_0 + \rho_0\,f(z) + \rho_c^0\,g(z)\,, \tag{10}$$

with

$$f(z) = \frac{\nu}{1-\nu}\left[(1+z)^\zeta - 1\right]\,, \tag{11}$$

$$g(z) = -\frac{\kappa(1+3\alpha)}{2(\zeta-2)} z(z+2) + \nu\kappa \frac{(1+z)^\zeta - 1}{(1-\nu)(\zeta-2)}. \tag{12}$$

Here ρ_0, ρ_c^0, Λ_0 and H_0 are respectively the matter-radiation energy density, critical density, CC and Hubble parameter at present, and we have used the following dimensionless coefficients:

$$\nu = \frac{\sigma M^2}{12\pi M_P^2}, \qquad \zeta = 3(1-\nu)(\alpha+1), \qquad \kappa = -2\nu\Omega_K^0. \tag{13}$$

It should be clear that there is only one single independent parameter in the model: ν. In the limit $\nu \to 0$ we recover the standard result for $\rho(z)$ with constant CC (see, e.g. [15]). In order to avoid confusion, let us note that the above solutions for $\rho(z;\nu)$ and $\Lambda(z;\nu)$ have no singularity in the limits $\zeta = 2$ and $\nu = 1$. It is easy to see that the function $g(z)$ differs from zero only for the cases of non-vanishing spatial curvature $\kappa \neq 0$, while another function $f(z)$ remains significant even for the flat space case $\kappa = 0$. Furthermore, in the limit $\nu \to 0$, $\Lambda(z;\nu)$ just becomes the z-independent standard CC.

The formulas above represent the universal solution at low energies, when all massive particles decouple according to (3). In order to understand the limits of applicability of these solutions, let us notice that when the temperature of the radiation energy density achieves the Fermi scale $T \sim M_F \simeq 293\,GeV$, the value of the Hubble parameter is just $H(M_F) \equiv 1.2 \times 10^{-4}\,eV$, unable to excite even the lightest neutrino. However, the most interesting scales are much smaller. Let us first consider the nucleosynthesis epoch when the radiation dominates over the matter, and derive the restriction on the parameter ν. In the RD regime, the solution for the density (9) can be rewritten in terms of the temperature

$$\rho_R(T) = \frac{\pi^2 g_*}{30}\left(r^{-\nu}T\right)^4 - \frac{\nu}{1-2\nu}\left[r^{4(1-\nu)} - r^2\right]\Omega_K^0 \rho_c^0, \tag{14}$$

where $r \equiv T/T_0$, $T_0 \simeq 2.75\,K = 2.37 \times 10^{-4}\,eV$ is the present CMB temperature, $g_* = 2$ for photons and $g_* = 3.36$ if we take the neutrinos into account, but this difference has no importance for the present considerations. It is easy to see that the size of the parameter ν gets restricted, because for $\nu \geq 1$ the density of radiation (in the flat case) would be the same or even below the one at the present universe. On the other hand near the nucleosynthesis time we have $T \gg T_0$, and so the corresponding value of the CC is

$$\Lambda_R(T) \simeq \frac{\nu}{1-\nu}\frac{\pi^2 g_*}{30}\left(r^{-\nu}T\right)^4 - \frac{\nu}{1-2\nu}\left[\frac{\nu}{1-\nu}r^{4(1-\nu)} - r^2\right]\Omega_K^0 \rho_c^0. \tag{15}$$

Hence, in order not to be ruled out by the nucleosynthesis, the ratio of the CC and the energy density at that time has to satisfy

$$|\Lambda_R / \rho_R| \simeq |\nu/(1-\nu)| \simeq |\nu| \ll 1. \tag{16}$$

A nontrivial range could e.g. be $0 < |\nu| \leq 0.1$. Both signs of ν are in principle allowed provided the absolute value satisfies the previous constraint. Let us notice that, in view of the definition (13), the condition $\nu \ll 1$ also means that $M \lesssim M_P$. Hence, the nucleosynthesis constraint coincides with our general will to remain in the framework of the effective approach. It is remarkable that
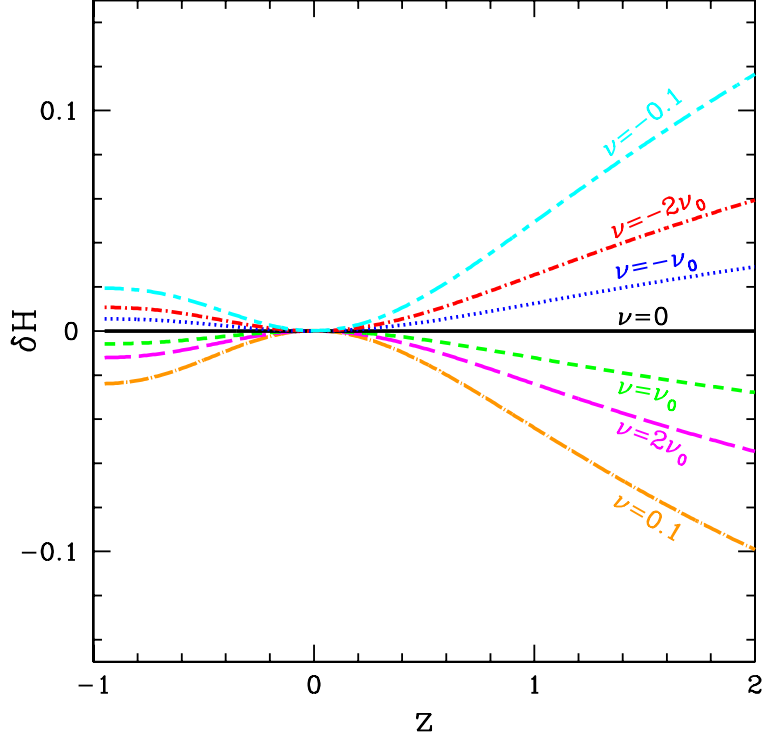
Figure 1: Relative deviation $\delta H(z; \nu)$, Eq. (20), of the Hubble parameter versus the redshift for various values of the single free parameter ν in flat space with $\Omega_M^0 = 0.3$ and $\Omega_\Lambda^0 = 0.7$.

the two constraints which come from very different considerations, lead to the very same restriction on the unique free parameter of the model. The canonical choice $M^2 = M_P^2$, corresponds to

$$|\nu| = \nu_0 \equiv \frac{1}{12\,\pi} \simeq 2.6 \times 10^{-2} \tag{17}$$

which certainly satisfies $\nu \ll 1$. Since the value of the parameter ν is small, all the effects of the renormalization group running of the CC will be approximately linear in ν.

Some phenomenological Implications.

Let us now turn to the "recent" Universe characterized by the redshift interval $0 < z \lesssim 2$, and let us evaluate some cosmological parameters which can be, in principle, extracted from the future observations, say by the SNAP project [4]. The first relevant exponent is the relative deviation δ_Λ of the CC from the constant value Λ_0. Then, using our solution (10), we obtain (expanding at first order in ν)

$$\Lambda(z) \simeq \Lambda_0 + \nu\, \rho_M^0 \left[(1+z)^3 - 1 \right] . \tag{18}$$

What about the numerical effect? The relative correction to the CC at redshift z is given by

$$\delta_\Lambda \equiv \frac{\Lambda(z; \nu) - \Lambda_0}{\Lambda_0} = \nu\, \frac{\Omega_M^0}{\Omega_\Lambda^0} \left[(1+z)^3 - 1 \right] . \tag{19}$$

Let us take flat space with $\Omega_M^0 = 0.3$, $\Omega_\Lambda^0 = 0.7$, and the value $\nu = \nu_0$ defined in (17). For $z = 1.5$ (reachable by SNAP [4]) we find $\delta_\Lambda = 16.3\%$, and so one that should be perfectly measurable by
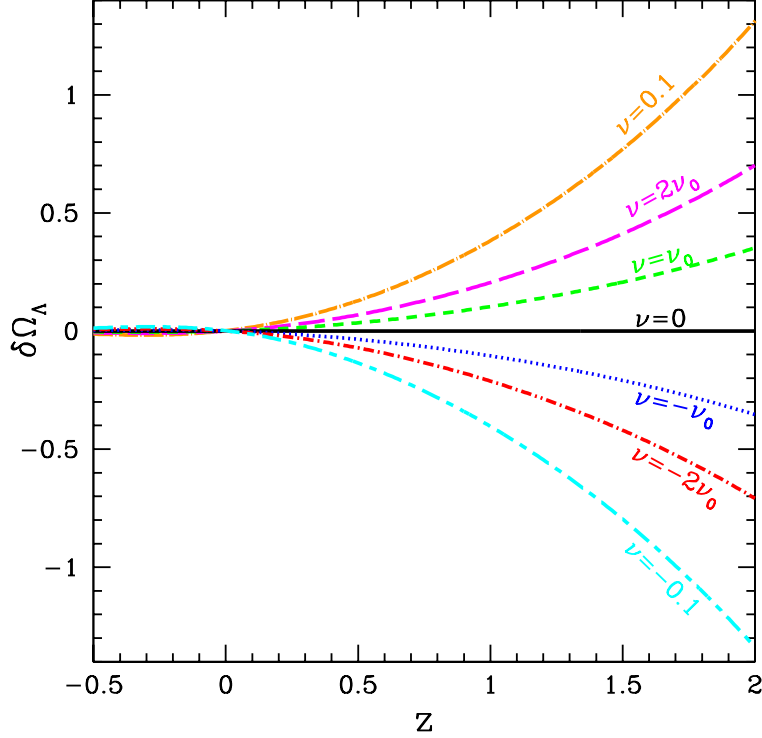
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Figure 2: Relative deviation $\delta\Omega_\Lambda(z;\nu)$, Eq.(21), of the cosmological constant parameter versus the redshift for the same cases as in Fig. 1.

SNAP. For larger values of ν the test can be much more efficient of course. For instance, values of order $\nu = 0.1$ are still perfectly tenable, in which case the previous correction would be as large as $\gtrsim 60\%$! In general, the strong cubic z-dependence in $\delta_\Lambda(z;\nu)$ should manifest itself in the future CC observational experiments where the range $z \gtrsim 1$ will be tested. It is important to emphasize that ν is the unique arbitrary parameter of this model for a variable CC. Therefore, the experimental verification of the above formula must consist in: *i)* pinning down the sign and value of the parameter ν; and *ii)* fitting that formula to the experimental data.

Next we present the relative deviation of the Hubble parameter $H(z,\nu)$ with respect to the conventional one $H(z,\nu = 0)$. At first order in ν,

$$\delta H(z;\nu) \equiv \frac{H(z;\nu) - H(z;0)}{H(z;0)}$$
$$\simeq -\frac{1}{2}\nu\,\Omega_M^0\,\frac{1 + (1+z)^3\,[3\ln(1+z) - 1]}{1 + \Omega_M^0\,[(1+z)^3 - 1]}\,. \tag{20}$$

Equation (20) gives the leading quantum correction to the Hubble parameter (7) when the renormalization effects in (9) and (10) are taken into account. Notice that $\delta H(0;\nu) = 0$, because for all ν we have the same initial conditions. Then for $z \neq 0$ we have e.g. $\delta H(1.5;\nu_0) \simeq -2\%$ and $\delta H(2;\nu_0) \simeq -3\%$. For higher ν we get sizeable effects: $\delta H(z;0.1) \simeq -8\%$ and -10% for $z = 1.5$ and $z = 2$ respectively. See Fig. 1 for the detailed numerical evaluation.

Although the induced corrections on H are not very high, the relative deviation of the renormalized cosmological constant parameter $\Omega_\Lambda(z;\nu) = 8\pi\,G\Lambda(z;\nu)/3H^2(z;\nu)$ with respect to the

standard one turns out to be much more significant. We get at leading order in ν,

$$\delta\Omega_\Lambda(z;\nu) \equiv \frac{\Omega_\Lambda(z;\nu) - \Omega_\Lambda(z;0)}{\Omega_\Lambda(z;0)} \tag{21}$$

$$\simeq \nu \left[\frac{\Omega_M^0 (1+z)^3 - 1}{\Omega_\Lambda^0} + \frac{1 + 3\,\Omega_M^0 (1+z)^3 \ln(1+z)}{\Omega_\Lambda^0 + \Omega_M^0 (1+z)^3} \right] .$$

Again $\delta\Omega_\Lambda(0;\nu) = 0$, as it should. Moreover, the deviation has the two expected limits for the infinite past and future, viz. $\delta\Omega_\Lambda(\infty;\nu) = \infty$ and $\delta\Omega_\Lambda(-1;\nu) = 0$. The numerical evaluation of (21) is given in Fig. 2. Notice that even for ν as small as ν_0, (17), there is a sizeable 20% increase of Ω_Λ at redshift $z = 1.5$ – reachable by SNAP. For $\nu = 2\nu_0$ the increase at $z = 1.5$ is huge, $\sim 40\%$. For $\nu < 0$ the effects go in the opposite direction. For fixed ν, the larger is z the larger are the quantum effects. Thus if $\nu \simeq \nu_0$ and some experiment in the future can reach the far $z = 2$ region with enough statistics, then the effects on $\Omega_\Lambda(z)$ and the Hubble parameter can be dramatic. If they are seen, this model can provide an explanation for them. At present Ω_Λ has been determined at roughly 10% from both supernovae and CMB measurements, and in the future SNAP will clinch Ω_Λ to within ± 0.05 [4]. The previous numbers show that for $z \gtrsim 1$ the cosmological quantum corrections can be measured already for a modest $\nu \gtrsim 10^{-2}$. A complete numerical analysis of this kind of FLRW models, including both the flat and curved space cases, together with a detailed comparison with the present and future Type Ia supernovae data, will be presented elsewhere [16].

Conclusions

To summarize, we have presented a semiclassical FLRW type of cosmological model based on a running CC with the scale $\mu = H$. We have shown that if the decoupling quantum effects on Λ have the usual form as for the massive fields, then we can get a handle on problem II by giving an alternative answer to problem III. In fact, we can explain the variation of the CC at low energies without resorting to any scalar field. The CC is mainly driven, without fine tuning, by the "relic" quantum effects from the physics of the highest available scale (the Planck scale), and its value naturally lies in the acceptable range. It is remarkable that all relevant information about the unknown world of the high energy physics is accumulated into a single parameter ν. Finally, we have shown that the next generation of supernovae experiments, like SNAP, should be sensitive to ν within its allowed range. A non-vanishing value of ν produces a cubic dependence of the CC on z at high redshift, which should be well measurable by that experiment, if it is really there. Therefore an efficient check of this alternative framework, which might be the effective behavior common to a large class of high energy theories, can in principle be performed in the near future.

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