ARTICLE IN PRESS
JID:YJCSS AID:2701 /FLA [m3G; v 1.96; Prn:20/05/2013; 12:47] P.1 (1-20)
Journal of Computer and System Sciences ••• (••••) •••–•••

Contents lists available at SciVerse ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

Displacement logic for anaphora ☆

Glyn Morrill [a,*], Oriol Valentín [b]

[a] Departament de Llenguatges i Sistemes Informàtics, Universitat Politècnica de Catalunya, Spain
[b] Barcelona Media, Universitat Pompeu Fabra, Spain

ARTICLE INFO

Article history:
Received 31 October 2010
Received in revised form 24 November 2011
Accepted 4 April 2013
Available online xxxx

Keywords:
Anaphora
Binding principles
Categorial logic
Cut-elimination
Displacement calculus
Negation as failure

ABSTRACT

The displacement calculus of Morrill, Valentín and Fadda (2011) [25] aspires to replace the calculus of Lambek (1958) [13] as the foundation of categorial grammar by accommodating intercalation as well as concatenation while remaining free of structural rules and enjoying Cut-elimination and its good corollaries. Jäger (2005) [11] proposes a type logical treatment of anaphora with syntactic duplication using limited contraction. Morrill and Valentín (2010) [24] apply (modal) displacement calculus to anaphora with lexical duplication and propose extension with a negation as failure in conjunction with additives to capture binding conditions. In this paper we present an account of anaphora developing characteristics and employing machinery from both of these proposals.

1. Introduction

Categorial grammar develops logical syntax, semantics and processing (see Moortgat [18,19], Morrill [26,27], Carpenter [3], Jäger [11]). Syntactically, grammatical categories or types are formulas of a non-commutative logic which reduces grammaticality to theoremhood. Semantically, a grammatical derivation or proof has a reading as an intuitionistic proof and hence a typed lambda term under the Curry–Howard correspondence, and this composes a logical semantic sentence meaning out of the lexical semantics of words represented by higher-order terms. Computationally, the logical grammar architecture is implemented by a parser/theorem-prover under the parsing-as-deduction paradigm.

The original foundation for such logical categorial grammar was the logic of concatenation of Lambek [13] which, however, has had a varied history. The calculus was largely lost in the wake of the tidal wave of transformational grammar until it was rediscovered in the 1980s, when it enjoyed a renaissance. At the end of the 1990s its founder pursued an alternative direction (see Lambek [15,16]), but one which retained concatenation-centricity while no longer maintaining the Curry–Howard categorial semantics. The former feature is what we consider the main shortcoming of Lambek calculus: that as a logic of concatenation it can capture some discontinuities, but only when these are peripheral — a specificity uncharacteristic of natural grammar.

☆ The research reported in the present paper was supported by DGICYT project SESAAME-BAR (TIN2008-06582-C03-01). We thank two anonymous JCSS reviewers for valuable comments and suggestions.
* Corresponding author.
E-mail addresses: morrill@lsi.upc.edu (G. Morrill), oriol.valentin@upf.edu (O. Valentín).
URL: http://www-lsi.upc.edu/~morrill/ (G. Morrill).

0022-0000/$ – see front matter © 2013 Published by Elsevier Inc.
http://dx.doi.org/10.1016/j.jcss.2013.05.006

The displacement calculus of Morrill, Valentín and Fadda [25] aspires to amend this shortcoming of the Lambek calculus as a foundation for logical categorial grammar by accommodating intercalation as well as concatenation while preserving the good technical properties of the Lambek calculus such as being free of structural rules and enjoying Cut-elimination and its corollaries: the subformula property, decidability and the finite reading property.[1] To support discontinuity that paper uses a novel kind of sequent calculus; Cut-elimination is proved and the paper illustrates a range of linguistic applications including anaphora. Morrill and Valentín [24] develop the treatment of anaphora further with modality for locality (see Morrill [21]) and negation as failure in conjunction with additives, for the binding principles of Chomsky [4].

In anaphora a pronoun shares its interpretation with an antecedent, so that semantic duplication is required. This duplication could be syntactically driven, as in the account with limited contraction of Jäger [11] (see also Jacobson [10] and Hepple [9]), or lexical, as in the account of Morrill, Valentín and Fadda [25] and Morrill and Valentín [24] (see also Morrill [22]). It is difficult to do justice to the intricacies of the decades of research that have been devoted to the testing and refining of anaphoric principles; still, in this paper we give a first approximation treatment of anaphora employing features from both of these options, and using as machinery displacement calculus with limited contraction, modality, additives and negation as failure. We prove Cut-elimination for the negation free fragment; it is not appropriate to admit Cut in the logic with the negation as failure, which is non-monotonic.

In Section 2 we define the displacement calculus **D** and its extension **DCA**$\Box$ with (a version of) limited contraction, additives and S4 modality, and we show Cut-elimination. In Section 3 we extend this with negation as failure. In Section 4 we review categorial semantics for the displacement logic. In Section 5 we present the treatment of anaphora; this is exemplified in Section 6. We conclude in Section 7.

2. Displacement calculus and extensions

2.1. The displacement calculus D

Let a *vocabulary* be a set including a placeholder symbol 1 called a *separator* or *marker*. We define the *sort* $\sigma(s)$ of a string s over the vocabulary as the number of placeholders it contains. For each natural i we define the *sort domain* L_i as the set of all strings containing i placeholders:

$$L_i = \{s \mid \sigma(s) = i\} \tag{1}$$

The concatenation $s_1 + s_2$ of a string s_1 of sort i with a string s_2 of sort j is a string of sort $i + j$, thus we have the functionality $+ : L_i, L_j \to L_{i+j}$. Note that concatenation is associative and that the empty string, which we notate 0, is a left and right identity for concatenation. In addition to concatenation, we define on (marked) strings two operations of intercalation or 'wrap'. Where α and β are strings and α contains at least one placeholder, we define the *leftmost wrap* of α around β, $\alpha \times_{\rangle} \beta$, as the result of replacing the leftmost placeholder in α by β, and we define the *rightmost wrap* of α around β, $\alpha \times_{\langle} \beta$, as the result of replacing the rightmost placeholder in α by β. For example:

$$(\mathbf{before} + 1 + \mathbf{left} + 1 + \mathbf{slept}) \times_{\langle} (\mathbf{the} + \mathbf{man}) = \mathbf{before} + 1 + \mathbf{left} + \mathbf{the} + \mathbf{man} + \mathbf{slept} \tag{2}$$

Thus where $k \in \{\rangle, \langle\}$ we have the functionalities $\times_k : L_{i+1}, L_j \to L_{i+j}$. Note that both leftmost and rightmost wraps are associative, and that in the same way that the empty string is a left and right identity for concatenation, the marker 1 is a left and right identity for leftmost and rightmost wrap. A vocabulary induces an ω-sorted algebra as follows:

$$(\{L_i\}_{i \in \mathcal{N}}, +, \times_{\rangle}, \times_{\langle}, 0, 1) \tag{3}$$

We call this a *displacement algebra*. A displacement algebra satisfies the following algebraic laws,[2] where $k \in \{\rangle, \langle\}$:

$$\begin{array}{lll} s_1 + (s_2 + s_3) = (s_1 + s_2) + s_3 & s_1 \times_k (s_2 \times_k s_3) = (s_1 \times_k s_2) \times_k s_3 & \text{associativity} \\ 0 + s = s & 1 \times_k s = s & \text{left identity} \\ s + 0 = s & s \times_k 1 = s & \text{right identity} \end{array} \tag{4}$$

The displacement calculus **D** is a logic of marked strings which has continuous connectives $\{\backslash, \bullet, /\}$ defined by residuation with respect to concatenation and discontinuous connectives $\{\downarrow_k, \odot_k, \uparrow_k\}_{k \in \{\rangle, \langle\}}$ defined by residuation with respect to leftmost and rightmost intercalation. The types of **D** are sorted into types $\mathcal{F}_i$ of sort i interpreted as sets of strings of sort i as shown in Fig. 1 where $k \in \{\rangle, \langle\}$. Where A is a type, sA is its sort. We shall optionally omit from connectives the subscript $\rangle$ for leftmost wrap.

[1] As we shall see, displacement calculus has a single placeholder symbol and employs as syntactic operations one step replacements of this symbol. By contrast, the lambda grammar of Muskens [28] and abstract categorial grammar of de Groote [8] have unboundedly many variable symbols and employ as the syntactic operation beta-reduction, i.e. the reflexive and transitive closure of beta-conversion.

[2] It satisfies additionally some further laws of mixed association and mixed permutation, cf. [29].

$\mathcal{F}_j ::= \mathcal{F}_i \backslash \mathcal{F}_{i+j}$	$[A\backslash C] = \{s_2 \mid \forall s_1 \in [A],\ s_1+s_2 \in [C]\}$	under
$\mathcal{F}_i ::= \mathcal{F}_{i+j}/\mathcal{F}_j$	$[C/B] = \{s_1 \mid \forall s_2 \in [B],\ s_1+s_2 \in [C]\}$	over
$\mathcal{F}_{i+j} ::= \mathcal{F}_i \bullet \mathcal{F}_j$	$[A \bullet B] = \{s_1+s_2 \mid s_1 \in [A]\ \&\ s_2 \in [B]\}$	product
$\mathcal{F}_0 ::= I$	$[I] = \{0\}$	product unit
$\mathcal{F}_j ::= \mathcal{F}_{i+1} \downarrow_k \mathcal{F}_{i+j}$	$[A \downarrow_k C] = \{s_2 \mid \forall s_1 \in [A],\ s_1 \times_k s_2 \in [C]\}$	infix
$\mathcal{F}_{i+1} ::= \mathcal{F}_{i+j} \uparrow_k \mathcal{F}_j$	$[C \uparrow_k B] = \{s_1 \mid \forall s_2 \in [B],\ s_1 \times_k s_2 \in [C]\}$	circumfix
$\mathcal{F}_{i+j} ::= \mathcal{F}_{i+1} \odot_k \mathcal{F}_j$	$[A \odot_k B] = \{s_1 \times_k s_2 \mid s_1 \in [A]\ \&\ s_2 \in [B]\}$	wrap
$\mathcal{F}_1 ::= J$	$[J] = \{1\}$	wrap unit

Fig. 1. Types of the displacement calculus **D** and their interpretation.

The set $\mathcal{O}$ of *configurations* is defined as follows, where Λ denotes the empty configuration and 1 is the metalinguistic marker:

$$\mathcal{O} ::= \Lambda \mid 1 \mid \mathcal{F}_0 \mid \mathcal{F}_{i+1}\{\underbrace{\mathcal{O} : \cdots : \mathcal{O}}_{i+1\ \mathcal{O}\text{'s}}\} \mid \mathcal{O}, \mathcal{O} \tag{5}$$

$A\{\Gamma_1 : \cdots : \Gamma_n\}$ interpreted syntactically is formed by strings $s_0+t_1+s_1+\cdots+s_{n-1}+t_n+s_n$ where $s_0+1+s_1+\cdots+s_{n-1}+1+s_n \in A$ and $t_1 \in \Gamma_1, \ldots, t_n \in \Gamma_n$. The *figure* or *vector* $\vec{A}$ of a type A is defined by:

$$\vec{A} = \begin{cases} A & \text{if the sort of } A \text{ is } 0 \\ A\{\underbrace{1 : \cdots : 1}_{sA\ 1\text{'s}}\} & \text{if the sort of } A \text{ is greater than } 0 \end{cases} \tag{6}$$

The sort of a configuration is the number of metalinguistic markers it contains. Where Δ is a configuration of sort $i > 0$ and $\Gamma_1, \ldots, \Gamma_i$ are configurations, the *fold* $\Delta \otimes \langle \Gamma_1, \ldots, \Gamma_i \rangle$ is the result of simultaneously replacing the successive placeholders in Δ by $\Gamma_1, \ldots, \Gamma_i$ respectively.

Where Δ and Γ are configurations and Δ is of sort $i > 0$, $\Delta|_{\rangle}\Gamma$ abbreviates

$$\Delta \otimes \langle \Gamma, \underbrace{1, \ldots, 1}_{i-1\ 1\text{'s}} \rangle \tag{7}$$

i.e. $\Delta|_{\rangle}\Gamma$ is the configuration which is the result of replacing the leftmost metalinguistic marker in Δ by Γ; and $\Delta|_{\langle}\Gamma$ abbreviates

$$\Delta \otimes \langle \underbrace{1, \ldots, 1}_{i-1\ 1\text{'s}}, \Gamma \rangle \tag{8}$$

i.e. $\Delta|_{\langle}\Gamma$ is the configuration which is the result of replacing the rightmost metalinguistic marker in Δ by Γ.[3]

The standard distinguished occurrence notation $\Delta(\Gamma)$ indicates a distinguished occurrence of Γ with external context Δ. Here, to deal with discontinuity, the *distinguished hyperoccurrence* notation $\Delta\langle\Gamma\rangle$ abbreviates $\Delta_0(\Gamma \otimes \langle \Delta_1, \ldots, \Delta_i \rangle)$, i.e. a potentially discontinuous distinguished occurrence of Γ with external context Δ_0 *and* internal contexts $\Delta_1, \ldots, \Delta_i$.

A *sequent* $\Gamma \Rightarrow A$ for the calculus of displacement **D** comprises an antecedent configuration Γ of sort i and a succedent type A of sort i. The sequent calculus for **D** is as shown in Fig. 2, where $k \in \{\rangle, \langle\}$.[4] It is the vectorial and distinguished hyperoccurrence notational devices which enable sequent calculus for displacement to be presented on the model of multimodal type logical grammar [19], but without any structural rules, and with $|_{\rangle}$ and $|_{\langle}$ as defined operations, not structural connectors: the only structural connector is the comma for concatenation, so that **D** has multimodal types but retains unimodal sequents.

2.2. Extension with limited contraction, additives and modality

The Lambek calculus is free of structural rules but anaphora involves duplication of antecedent semantics. Jäger [11] extends the Lambek calculus with limited contraction to provide an account of anaphora with syntactic duplication. Here we employ a very slight variant of this in the context of the displacement calculus. Limited contraction is for a binary type-constructor $|$ such that $B|A$ signifies an expression of type B containing a free anaphor of type A (cf. Jacobson [10], who writes B^A). We extend the types of the displacement calculus as follows:

$$\mathcal{F}_{i+j} ::= \mathcal{F}_{i+j}|\mathcal{F}_j \tag{9}$$

[3] Thanks to Philippe de Groote for pointing out that leftmost and rightmost replacements are special cases of fold.

[4] Note that by IL I can be inserted anywhere ($\vec{I}$ could have been written I since it is of sort 0); likewise by JL J can be wrapped any number of times around a separator.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

$$\frac{}{\overrightarrow{A} \Rightarrow A}\, id \qquad \frac{\Gamma \Rightarrow A \quad \Delta\langle\overrightarrow{A}\rangle \Rightarrow B}{\Delta\langle\Gamma\rangle \Rightarrow B}\, Cut$$

$$\frac{\Gamma \Rightarrow A \quad \Delta\langle\overrightarrow{C}\rangle \Rightarrow D}{\Delta\langle\Gamma, \overrightarrow{A\backslash C}\rangle \Rightarrow D}\, \backslash L \qquad \frac{\overrightarrow{A}, \Gamma \Rightarrow C}{\Gamma \Rightarrow A\backslash C}\, \backslash R$$

$$\frac{\Gamma \Rightarrow B \quad \Delta\langle\overrightarrow{C}\rangle \Rightarrow D}{\Delta\langle\overrightarrow{C/B}, \Gamma\rangle \Rightarrow D}\, /L \qquad \frac{\Gamma, \overrightarrow{B} \Rightarrow C}{\Gamma \Rightarrow C/B}\, /R$$

$$\frac{\Delta\langle\overrightarrow{A}, \overrightarrow{B}\rangle \Rightarrow D}{\Delta\langle\overrightarrow{A \bullet B}\rangle \Rightarrow D}\, \bullet L \qquad \frac{\Gamma_1 \Rightarrow A \quad \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow A \bullet B}\, \bullet R$$

$$\frac{\Delta\langle\Lambda\rangle \Rightarrow A}{\Delta\langle\overrightarrow{I}\rangle \Rightarrow A}\, IL \qquad \frac{}{\Lambda \Rightarrow I}\, IR$$

$$\frac{\Gamma \Rightarrow A \quad \Delta\langle\overrightarrow{C}\rangle \Rightarrow D}{\Delta\langle\Gamma|_k\overrightarrow{A\downarrow_k C}\rangle \Rightarrow D}\, \downarrow_k L \qquad \frac{\overrightarrow{A}|_k\Gamma \Rightarrow C}{\Gamma \Rightarrow A\downarrow_k C}\, \downarrow_k R$$

$$\frac{\Gamma \Rightarrow B \quad \Delta\langle\overrightarrow{C}\rangle \Rightarrow D}{\Delta\langle\overrightarrow{C\uparrow_k B}|_k\Gamma\rangle \Rightarrow D}\, \uparrow_k L \qquad \frac{\Gamma|_k\overrightarrow{B} \Rightarrow C}{\Gamma \Rightarrow C\uparrow_k B}\, \uparrow_k R$$

$$\frac{\Delta\langle\overrightarrow{A}|_k\overrightarrow{B}\rangle \Rightarrow D}{\Delta\langle\overrightarrow{A \odot_k B}\rangle \Rightarrow D}\, \odot_k L \qquad \frac{\Gamma_1 \Rightarrow A \quad \Gamma_2 \Rightarrow B}{\Gamma_1|_k\Gamma_2 \Rightarrow A \odot_k B}\, \odot_k R$$

$$\frac{\Delta\langle 1\rangle \Rightarrow A}{\Delta\langle\overrightarrow{J}\rangle \Rightarrow A}\, JL \qquad \frac{}{1 \Rightarrow J}\, JR$$

Fig. 2. Hypersequent calculus for **D**.

We assume rules as follows, where the semicolon separates disjoint hyperoccurrences which may be consistently in any order left-to-right[5]:

$$\frac{\Gamma \Rightarrow A \quad \Delta\langle\overrightarrow{A}; \overrightarrow{B}\rangle \Rightarrow D}{\Delta\langle\Gamma; \overrightarrow{B|A}\rangle \Rightarrow D}\, |L \qquad \frac{\Gamma\langle\overrightarrow{B_0}; \ldots; \overrightarrow{B_n}\rangle \Rightarrow D}{\Gamma\langle\overrightarrow{B_0|A}; \ldots; \overrightarrow{B_n|A}\rangle \Rightarrow D|A}\, |R \tag{10}$$

We call **DC** the extension of **D** with this version of limited contraction.

The displacement calculus is a multiplicative system in the terminology of linear logic [7]. We call **DA** the extension of this with additives [14,20,12]. In the sorting and sequent regime of the displacement calculus these are as follows:

$$\mathcal{F}_i ::= \mathcal{F}_i \,\&\, \mathcal{F}_i \mid \mathcal{F}_i \oplus \mathcal{F}_i \tag{11}$$

$$\frac{\Gamma\langle\overrightarrow{A}\rangle \Rightarrow C}{\Gamma\langle\overrightarrow{A \,\&\, B}\rangle \Rightarrow C}\, \& L_1 \qquad \frac{\Gamma\langle\overrightarrow{B}\rangle \Rightarrow C}{\Gamma\langle\overrightarrow{A \,\&\, B}\rangle \Rightarrow C}\, \& L_2$$

$$\frac{\Gamma \Rightarrow A \quad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \,\&\, B}\, \& R$$

$$\frac{\Gamma\langle\overrightarrow{A}\rangle \Rightarrow C \quad \Gamma\langle\overrightarrow{B}\rangle \Rightarrow C}{\Gamma\langle\overrightarrow{A \oplus B}\rangle \Rightarrow C}\, \oplus L$$

$$\frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A \oplus B}\, \oplus L_1 \qquad \frac{\Gamma \Rightarrow B}{\Gamma \Rightarrow A \oplus B}\, \oplus L_2 \tag{12}$$

[5] Jäger [11] has only $|L$ (limited contraction) with the antecedent preceding the anaphor, giving rise to backward anaphora only; our variant allows also forward anaphora (cataphora).

We call $\mathbf{D}\Box$ the extension of $\mathbf{D}$ with S4 modality. In the sorting and sequent regime of the displacement calculus this is as follows, where $\Box\Gamma$ signifies a configuration all the types of which have $\Box$ as the main connective.

$$\mathcal{F}_i ::= \Box\mathcal{F}_i \tag{13}$$

$$\dfrac{\Gamma\langle\vec{A}\rangle \Rightarrow B}{\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow B}\Box L \qquad \dfrac{\Box\Gamma \Rightarrow A}{\Box\Gamma \Rightarrow \Box A}\Box R \tag{14}$$

We call $\mathbf{DCA}\Box$ the extension of $\mathbf{D}$ with the limited contraction, additives and S4 modality.

2.3. Remarks on Cut-elimination for DCA□

A few words on technical details and notation. The notation already used $\Delta\langle\Gamma_1;\dots;\Gamma_n\rangle$ represents a hypercontext Δ with n (hyper)holes ($n > 0$) each one substituted by a configuration or a hypercontext. We define the weight of a type as its number of connective occurrences. The weight of an atomic type (of arbitrary sort) is 0. The strategy of the proof of Cut-elimination follows Lambek [13] with the notion of Cut degree which is based on the weight of types. We transform variations in such a way that Cuts are removed or substituted by other Cuts which are, crucially, of lesser degree, while preserving the endsequent. In this way, since no Cut degree is negative the transformation procedure always yields a Cut-free proof in a finite number of steps. Cut-elimination for $\mathbf{D}$ is proved in the appendix of Morrill, Valentín and Fadda [25].

2.4. Cut-elimination for DC

Permutation conversions are standard and behave in the same way as for the other connectives. We consider the principal Cut cases which are not so standard. The reader should notice that our metanotation of sequents simplifies the way that Jäger [11] presents the principal Cut cases; as he remarks there are two possible cases of principal Cut for the connective $|$:

- The case where the minor premise of Cut is the right $|$ rule and the rule of the major premise of Cut is the left $|$ rule:

$$\dfrac{\dfrac{\Delta\langle\overrightarrow{D_1};\dots;\overrightarrow{D_n}\rangle \Rightarrow A}{\Delta\langle\overrightarrow{D_1|B};\dots;\overrightarrow{D_n|B}\rangle \Rightarrow A|B}|R \qquad \dfrac{\Gamma \Rightarrow B \quad \Theta\langle\vec{B};\vec{A}\rangle \Rightarrow C}{\Theta\langle\Gamma;\overrightarrow{A|B}\rangle \Rightarrow C}|L}{\Theta\langle\Gamma;\Delta\langle\overrightarrow{D_1|B};\dots;\overrightarrow{D_n|B}\rangle\rangle \Rightarrow C}Cut$$

$$\rightsquigarrow$$

$$\dfrac{\Gamma \Rightarrow B \qquad \dfrac{\begin{array}{c}\dfrac{\vec{B} \Rightarrow B \qquad \dfrac{\vec{B} \Rightarrow B \qquad \dfrac{\Delta\langle\overrightarrow{D_1};\dots;\overrightarrow{D_n}\rangle \Rightarrow A \quad \Theta\langle\vec{B};\vec{A}\rangle \Rightarrow C}{\Theta\langle\vec{B};\Delta\langle\overrightarrow{D_1};\dots;\overrightarrow{D_n}\rangle\rangle \Rightarrow C}Cut}{\Theta\langle\vec{B};\Delta\langle\overrightarrow{D_1|B};\dots;\overrightarrow{D_n}\rangle\rangle \Rightarrow C}|L}{}|L\\ \vdots\end{array}}{\Theta\langle\vec{B};\Delta\langle\overrightarrow{D_1|B};\dots;\overrightarrow{D_{n-1}|B};\overrightarrow{D_n}\rangle\rangle \Rightarrow C}|L}{\Theta\langle\Gamma;\Delta\langle\overrightarrow{D_1|B};\dots;\overrightarrow{D_n|B}\rangle\rangle \Rightarrow C}Cut$$

- The case where the rule of the minor premise of the Cut rule is a $|$ right rule and the rule of the major premise of the Cut rule is a $|$ right rule[6]:

$$\dfrac{\dfrac{\Delta\langle\overrightarrow{A_1};\dots;\overrightarrow{A_n}\rangle \Rightarrow B_i}{\Delta\langle\overrightarrow{A_1|C};\dots;\overrightarrow{A_n|C}\rangle \Rightarrow B_i|C}|R \qquad \dfrac{\Gamma\langle\overrightarrow{B_1};\dots;\overrightarrow{B_m}\rangle \Rightarrow D}{\Gamma\langle\overrightarrow{B_1|C};\dots;\overrightarrow{B_m|C}\rangle \Rightarrow D|C}|R}{\Gamma\langle\overrightarrow{B_1|C};\dots;\Delta\langle\overrightarrow{A_1|C};\dots;\overrightarrow{A_n|C}\rangle;\overrightarrow{B_{i+1}|C};\dots;\overrightarrow{B_m|C}\rangle \Rightarrow D|C}Cut$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\Delta\langle\overrightarrow{A_1};\dots;\overrightarrow{A_n}\rangle \Rightarrow B_i \quad \Gamma\langle\overrightarrow{B_1};\dots;\overrightarrow{B_m}\rangle \Rightarrow D}{\Gamma\langle\overrightarrow{B_1};\dots;\Delta\langle\overrightarrow{A_1};\dots;\overrightarrow{A_n}\rangle;\overrightarrow{B_{i+1}};\dots;\overrightarrow{B_m}\rangle \Rightarrow D}Cut}{\Gamma\langle\overrightarrow{B_1|C};\dots;\Delta\langle\overrightarrow{A_1|C};\dots;\overrightarrow{A_n|C}\rangle;\overrightarrow{B_{i+1}|C};\dots;\overrightarrow{B_m|C}\rangle \Rightarrow D|C}|R$$

[6] It must be remarked that this case of principal Cut is somewhat non-standard in the tradition of proof theory because, as observed by a referee, the so-called right rule for Jäger's connective '|' is not a proper dual of the left rule, since it introduces the $|$ on both sides of the sequent symbol.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

ARTICLE IN PRESS
JID:YJCSS AID:2701 /FLA [m3G; v 1.96; Prn:20/05/2013; 12:47] P.6 (1-20)

2.4.1. *Cut-elimination for* **DA**

See, for example, [5]. No special new issues arise in the context of sequent calculus for displacement.

2.4.2. *Cut-elimination for* **D□**

Where Γ denotes a configuration or hypercontext, $\Box\Gamma$ represents a configuration or hypercontext in which all the occurrences of types are modalized outermost. We sketch the proof for some cases:

- Principal Cut:

$$\cfrac{\cfrac{\Box\Gamma \Rightarrow A}{\Box\Gamma \Rightarrow \Box A}\,\Box R \qquad \cfrac{\Delta\langle\overrightarrow{A}\rangle \Rightarrow B}{\Delta\langle\overrightarrow{\Box A}\rangle \Rightarrow B}\,\Box L}{\Delta\langle\Box\Gamma\rangle \Rightarrow B}\,Cut$$

$$\rightsquigarrow$$

$$\cfrac{\Box\Gamma \Rightarrow A \qquad \Delta\langle\overrightarrow{A}\rangle \Rightarrow B}{\Delta\langle\Box\Gamma\rangle \Rightarrow B}\,Cut$$

- Permutation conversion:
  We permute the application of the logical rule and the Cut rule. We consider only a case which is a little bit more problematic than usual. Let us suppose that the last rule of the major premise of the Cut rule is an instance of the $\Box$ right rule:

$$\cfrac{\Delta \Rightarrow \Box A \qquad \cfrac{\Box\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow B}{\Box\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow \Box B}\,\Box R}{\Box\Gamma\langle\Delta\rangle \Rightarrow \Box B}\,Cut$$

  In this case the standard permutation of the logical rule with Cut does not necessarily work; consider:

$$\cfrac{\Delta \Rightarrow \Box A \qquad \Box\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow B}{\Box\Gamma\langle\Delta\rangle \Rightarrow B}\,Cut$$

  Now the right $\Box$ rule may not be applicable because Δ is not guaranteed to be fully modalized. The way to fix this consists of permuting the left rule of the minor premise of the Cut rule with the Cut rule as follows[7]:

$$\cfrac{\cfrac{\Delta^\star \Rightarrow \Box A}{\Delta \Rightarrow \Box A}\,\text{rule} \qquad \cfrac{\Box\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow B}{\Box\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow \Box B}\,\Box R}{\Box\Gamma\langle\Delta\rangle \Rightarrow \Box B}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\Delta^\star \Rightarrow \Box A \qquad \Box\Gamma\langle\overrightarrow{\Box A}\rangle \Rightarrow \Box B}{\Box\Gamma\langle\Delta^\star\rangle \Rightarrow \Box B}\,Cut}{\Box\Gamma\langle\Delta\rangle \Rightarrow \Box B}\,\text{rule}$$

3. Negation as failure

Binding theory has negative constraints. We would like then to incorporate negative information into our grammars. Given our type-logical approach, this forces us to account for negative information in the lexicon, namely in the types.

Since **DCA□** is a type logic, we are naturally driven to look for a new connective, namely a kind of negation. This is the kind of operator we need. But as we know, in the landscape of substructural logics there are a huge variety of negations. A negation which immediately comes to mind is the negation of linear logic. Given our intuitionistic regime, we could add to **DCA□** the constant $\bot$ and define negation in terms of one implication or several implications which are at our disposal in our type logic **DCA□**, namely the continuous implications $\{\backslash, /\}$ and the discontinuous implications $\{\downarrow_k, \uparrow_k\colon\ k \in \{\langle, \rangle\}\}$. $\bot$ would have the rules[8]:

[7] We suppose without loss of generality that the left rule which applies at the minor premise of the Cut is a unary rule; here 'rule' denotes any left unary rule. A binary rule would be quite similar.

[8] Here we leave aside the problem of the sort of $\bot$. The rule we present in (15) would entail of course that $\bot$ would have sort 0. But it would be natural in our setting to formulate constants $\bot_i$ $(i > 0)$ of arbitrary sort i. This is a possibility we do not explore here.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

$$\frac{}{\perp\ \Rightarrow}\perp L \qquad \frac{\Delta\ \Rightarrow}{\Delta\ \Rightarrow\ \perp}\perp R \tag{15}$$

Let us consider the case of the negation defined in terms of the continuous implication *under* $\backslash$:

$$\neg A \stackrel{def}{=} A\backslash\perp$$

Suppose that one wants to type a word as a functor X/n_{tr} which requires as argument a typed raised noun phrase, i.e. X/n_{tr} can combine with $s/(n\backslash s)$, but cannot combine with a simple noun phrase n. Using the additive conjunction & and the defined $\neg$, n_{tr} would be defined as follows:

$$n_{tr} := s/(n\backslash s)\ \&\ \neg n \tag{16}$$

We would expect then that:

$$\vdash s/(n\backslash s)\ \Rightarrow\ n_{tr}$$

$$\nvdash n\ \Rightarrow\ n_{tr}$$

However, let us see the case of:

$$\vdash s/(n\backslash s)\ \Rightarrow\ n_{tr}$$

Clearly, the last rule of the above sequent should be $\& R$:

$$\frac{s/(n\backslash s)\ \Rightarrow\ s/(n\backslash s) \quad s/(n\backslash s)\ \Rightarrow\ \neg n}{s/(n\backslash s)\ \Rightarrow\ n_{tr}}\ \& R \tag{17}$$

The first premise of the above derivation is obvious. Let us see the second one:

$$\frac{n, s/(n\backslash s)\ \Rightarrow\ \perp}{s/(n\backslash s)\ \Rightarrow\ \neg n}\backslash R$$

By Cut elimination (which we can assume, since we are using a standard constant from intuitionistic logic which we add to **DCA**□), a simple inspection at the sequent shows that:

$$\nvdash n, s/(n\backslash s)\ \Rightarrow\ \perp$$

It is clear then that a constructive negation does not play the role we want.

A reasonable alternative for the constructive linear negation is the one which Buszkowski [2] proposed in his paper *Categorial grammars with negative information*. Here the negation, which we denote $\neg_b$, is a primitive connective which has a boolean behaviour. More concretely, Buszkowski extends the Lambek calculus **L** with two axioms and a rule of transposition. The presentation of the calculus, which we denote $\mathbf{L}_{\neg_b}$ is categorical:

- Standard rules of residuation, the axiom rule, and the transitive rule of the arrow $\rightarrow$.
- Two axioms:

$$A \rightarrow \neg_b\neg_b A \quad \text{and} \quad \neg_b\neg_b A \rightarrow A \quad \text{for any type } A$$

- The rule of transposition:

$$\frac{A \rightarrow B}{\neg_b B \rightarrow \neg_b A}\text{trans}$$

The calculus $\mathbf{L}_{\neg_b}$ suffers some problems. A Gentzen sequent presentation is not known, and more importantly, the decidability of $\mathbf{L}_{\neg_b}$ is an open problem. Again, this is not the connective we want.

Morrill and Valentín [24] introduce into type logical categorial grammar a negation interpreted in the succedent as non-provability which is in fact a *negation as failure*. Negation as failure has been studied in the framework of autoepistemic logic (see for example [17]):

$$\frac{\nvdash \Gamma\ \Rightarrow\ A}{\Gamma\ \Rightarrow\ \neg A}\neg R \tag{18}$$

Thus for example, to express that *walk* is a non-third person present tense form we might assign to it a type such as $(\exists a N(a)\ \&\ \neg N(3(sg)))\backslash S$.

$$\begin{array}{rcll}
\Phi_\tau & ::= & C_\tau & \\
\Phi_\tau & ::= & V_\tau & \\
\Phi_\top & ::= & 0 & \\
\Phi_\tau & ::= & \Phi_{\tau_1+\tau_2} \to V_{\tau_1}.\Phi_\tau;\ V_{\tau_2}.\Phi_\tau & \text{case statement} \\
\Phi_{\tau+\tau'} & ::= & \iota_1\Phi_\tau & \text{first injection} \\
\Phi_{\tau'+\tau} & ::= & \iota_2\Phi_\tau & \text{second injection} \\
\Phi_\tau & ::= & \pi_1\Phi_{\tau\ \&\ \tau'} & \text{first projection} \\
\Phi_\tau & ::= & \pi_2\Phi_{\tau'\ \&\ \tau} & \text{second projection} \\
\Phi_{\tau\ \&\ \tau'} & ::= & (\Phi_\tau, \Phi_{\tau'}) & \text{ordered pair formation} \\
\Phi_\tau & ::= & (\Phi_{\tau'\to\tau}\ \Phi_{\tau'}) & \text{functional application} \\
\Phi_{\tau\to\tau'} & ::= & \lambda V_\tau \Phi_{\tau'} & \text{functional abstraction} \\
\Phi_\tau & ::= & {}^\vee\Phi_{\mathbf{L}\tau} & \text{extensionalization} \\
\Phi_{\mathbf{L}\tau} & ::= & {}^\wedge\Phi_\tau & \text{intensionalization}
\end{array}$$

Fig. 3. Syntax of terms for semantic representation.

As regards the Cut rule and negation as failure, note that by using them both together we would get undesirable derivations such as the following:

$$\dfrac{N \Rightarrow S/(N\backslash S) \qquad \dfrac{\nvdash S/(N\backslash S) \Rightarrow N}{S/(N\backslash S) \Rightarrow \neg N}\neg R}{N \Rightarrow \neg N}\mathit{Cut} \tag{19}$$

Adding the negation as failure (right) rule brings our categorial logic into the realms of non-monotonic reasoning where the transitivity of the consequence relation must be dropped. As we have seen the other connectives used in this paper, the displacement connectives, limited contraction, additives and S4 modality, enjoy Cut-elimination. But in the presence of negation as failure, the Cut rule must be considered not only no longer eliminable, but inadmissible. However, the subformula property holds of all the connectives used here: the sequent presentation is such that for every rule, the formula occurrences in the premises are always subformulas of those in the conclusion. Given this state of affairs, the Cut-free backward chaining sequent proof search space is finite and hence the categorial logic **DCA**□ plus negation as failure used in this paper is decidable.

Here we will use the negation only in the context of succedent $A \ \& \ \neg B$, which we represent by a synthetic difference operator. Synthetic connectives are defined connectives for which rules can be derived as if they were primitives. They serve to abbreviate. As we shall see in the next section, dropping the negation and maintaining the synthetic difference connective will, crucially, assign a Curry–Howard term to all the derivations of sequents in **DCA**□ plus the difference operator. Our account of anaphora will make essential use of the mentioned synthetic connective difference '−'.

4. Semantics

The set $\mathcal{T}$ of *semantic types* is defined on the basis of a set δ of *basic semantic types* as follows:

$$\mathcal{T} ::= \delta \mid \top \mid \mathcal{T}+\mathcal{T} \mid \mathcal{T}\,\&\,\mathcal{T} \mid \mathcal{T}\to\mathcal{T} \mid \mathbf{L}\mathcal{T} \tag{20}$$

A *semantic frame* comprises a non-empty set W of worlds and a family $\{D_\tau\}_{\tau\in\mathcal{T}}$ of non-empty *semantic type domains* such that:

$$\begin{array}{lll}
D_\top = \{0\} & & \text{singleton set} \\
D_{\tau_1+\tau_2} = D_{\tau_2} \uplus D_{\tau_1} & (\{1\}\times D_{\tau_1}) \cup (\{2\}\times D_{\tau_2}) & \text{disjoint union} \\
D_{\tau_1\ \&\ \tau_2} = D_{\tau_1} \times D_{\tau_2} & \{\langle m_1, m_2\rangle \mid m_1 \in D_{\tau_1}\ \&\ m_2 \in D_{\tau_2}\} & \text{Cartesian product} \\
D_{\tau_1\to\tau_2} = D_{\tau_2}^{D_{\tau_1}} & \text{the set of all functions from } D_{\tau_1} \text{ to } D_{\tau_2} & \text{functional exponentiation} \\
D_{\mathbf{L}\tau} = D_\tau^W & \text{the set of all functions from } W \text{ to } D_\tau & \text{functional exponentiation}
\end{array} \tag{21}$$

The sets Φ_τ of *terms* of type τ for each type τ are defined on the basis of sets C_τ of constants of type τ and enumerably infinite sets V_τ of variables of type τ for each type τ as shown in Fig. 3.

Given a semantic frame, a *valuation* f is a function mapping each constant of type τ into an element of D_τ, and an *assignment* g is a function mapping each variable of type τ into an element of D_τ. Where g is such, the update $g[x := m]$ is $(g - \{(x, g(x))\}) \cup \{(x, m)\}$. Relative to a valuation, an assignment g and a world $i \in W$, each term ϕ of type τ receives an interpretation $[\phi]^{g,i} \in D_\tau$ as shown in Fig. 4.

An occurrence of a variable x in a term is called *free* if and only if it does not fall within any part of the term of the form $x.\cdot$ or $\lambda x\cdot$; otherwise it is *bound* (by the closest $x.$ or λx within the scope of which it falls). The result $\phi[\psi_1/x, \ldots, \psi_n/x_n]$ of substituting terms $\psi_1, \ldots, \psi_n$ (of types $\tau_1, \ldots, \tau_n$) for variables $x_1, \ldots, x_n$ (of types $\tau_1, \ldots, \tau_n$) in a term ϕ is the result of simultaneously replacing by $\psi_1, \ldots, \psi_n$ every free occurrence of $x_1, \ldots, x_n$ respectively in ϕ. We say that ψ is *free for x in* ϕ

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

$$
\begin{aligned}
[a]^{g,i} &= f(a) \text{ for constant } a \in C_\tau \\
[x]^{g,i} &= g(x) \text{ for variable } x \in V_\tau \\
[0]^{g,i} &= 0 \\
[\phi \to x.\psi;\, y.\chi]^{g,i} &= \begin{cases} [\psi]^{g[x:=m],i} & \text{if } [\phi]^{g,i} = \langle 1, m\rangle \\ [\chi]^{g[y:=m],i} & \text{if } [\phi]^{g,i} = \langle 2, m\rangle \end{cases} \\
[\iota_1\phi]^{g,i} &= \langle 1, [\phi]^{g,i}\rangle \\
[\iota_2\phi]^{g,i} &= \langle 2, [\phi]^{g,i}\rangle \\
[\pi_1\phi]^{g,i} &= \mathbf{fst}([\phi]^{g,i}) \\
[\pi_2\phi]^{g,i} &= \mathbf{snd}([\phi]^{g,i}) \\
[(\phi, \psi)]^{g,i} &= \langle [\phi]^{g,i}, [\psi]^{g,i}\rangle \\
[(\phi\ \psi)]^{g,i} &= [\phi]^{g,i}([\psi]^{g,i}) \\
[\lambda x\phi]^{g,i} &= m \mapsto [\phi]^{g[x:=m],i} \\
[{}^\vee\phi]^{g,i} &= [\phi]^{g,i}(i) \\
[{}^\wedge\phi]^{g,i} &= j \mapsto [\phi]^{g,j}
\end{aligned}
$$

Fig. 4. Semantics of terms for semantic representation.

$$
\begin{aligned}
&\phi \to x.\psi;\, y.\chi = \phi \to z.(\psi[z/x]);\, y.\chi \\
&\text{if } z \text{ is not free in } \psi \text{ and is free for } x \text{ in } \psi \\
&\phi \to x.\psi;\, y.\chi = \phi \to x.\psi;\, z.(\chi[z/y]) \\
&\text{if } z \text{ is not free in } \chi \text{ and is free for } y \text{ in } \chi \\
&\lambda x\phi = \lambda y(\phi[y/x]) \\
&\text{if } y \text{ is not free in } \phi \text{ and is free for } x \text{ in } \phi
\end{aligned}
$$

α-conversion

$$
\begin{aligned}
&\iota_1\phi \to y.\psi;\, z.\chi = \psi[\phi/y] \\
&\text{if } \phi \text{ is free for } y \text{ in } \psi \text{ and modally free for } y \text{ in } \psi \\
&\iota_2\phi \to y.\psi;\, z.\chi = \chi[\phi/z] \\
&\text{if } \phi \text{ is free for } z \text{ in } \chi \text{ and modally free for } z \text{ in } \chi \\
&\pi_1(\phi, \psi) = \phi \\
&\pi_2(\phi, \psi) = \psi \\
&(\lambda x\phi\ \psi) = \phi[\psi/x] \\
&\text{if } \psi \text{ is free for } x \text{ in } \phi, \text{ and modally free for } x \text{ in } \phi \\
&{}^{\vee\wedge}\phi = \phi
\end{aligned}
$$

β-conversion

$$
\begin{aligned}
&(\pi_1\phi, \pi_2\phi) = \phi \\
&\lambda x(\phi\ x) = \phi \\
&\text{if } x \text{ is not free in } \phi \\
&{}^{\wedge\vee}\phi = \phi \\
&\text{if } \phi \text{ is modally closed}
\end{aligned}
$$

η-conversion

Fig. 5. Semantic conversion laws.

if and only if no variable occurrence in ψ becomes bound in $\phi[\psi/x]$ (i.e. if and only if there is no "accidental capture"). We say that a term is *modally closed* if and only if every occurrence of $^\vee$ occurs within the scope of an $^\wedge$. A modally closed term is denotationally invariant across worlds. We say that a term ψ is *modally free for x in* ϕ if and only if either ψ is modally closed, or no free occurrence of x in ϕ is within the scope of an $^\wedge$. The laws of conversion in Fig. 5 obtain; for the sake of brevity we omit the so-called commuting conversions for the case statement.

The definition of syntactic types and the semantic type map T sending syntactic types to semantic types is as shown in Fig. 6 for **DCA**□ with succedent difference. The definition distinguishes types with antecedent polarity (superscript $^\bullet$) and succedent polarity (superscript ^{o}); where p is a polarity, $\bar{p}$ is the opposite polarity. Some semantically labelled sequent rules (which are sufficient for our account of anaphora) of **DCA**□ are given in Fig. 7.[9] As we said in the previous section, all the derivations of sequents in **DCA**□ receive a Curry–Howard term:

Fact: Every derivation $\mathcal{D}$ of a provable sequent $\Delta \Rightarrow A$ in **DCA**□ receives a Curry–Howard term $\Phi_{\mathcal{D}}$ (22)

[9] The underscore in the negative subgoal of the difference right rule is an anonymous metavariable.

$$
\begin{array}{rcl@{\qquad}rcl}
\mathcal{F}_j^p & ::= & \mathcal{F}_i^{\bar{p}} \backslash \mathcal{F}_{i+j}^p & T(A\backslash C) & = & T(A) \to T(C) \\
\mathcal{F}_i^p & ::= & \mathcal{F}_{i+j}^p / \mathcal{F}_j^{\bar{p}} & T(C/B) & = & T(B) \to T(C) \\
\mathcal{F}_{i+j}^p & ::= & \mathcal{F}_i^p \bullet \mathcal{F}_j^p & T(A \bullet B) & = & T(A)\ \&\ T(B) \\
\mathcal{F}_0^p & ::= & I & T(I) & = & \top \\
\mathcal{F}_j^p & ::= & \mathcal{F}_{i+1}^{\bar{p}} \downarrow_k \mathcal{F}_{i+j}^p & T(A \downarrow_k C) & = & T(A) \to T(C) \\
\mathcal{F}_{i+1}^p & ::= & \mathcal{F}_{i+j}^p \uparrow_k \mathcal{F}_j^{\bar{p}} & T(C \uparrow_k B) & = & T(B) \to T(C) \\
\mathcal{F}_{i+j}^p & ::= & \mathcal{F}_{i+1}^p \odot_k \mathcal{F}_j^p & T(A \odot_k B) & = & T(A)\ \&\ T(B) \\
\mathcal{F}_1^p & ::= & J & T(J) & = & \top \\
\mathcal{F}_{i+j}^p & ::= & \mathcal{F}_{i+j}^p | \mathcal{F}_j^{\bar{p}} & T(B|A) & = & T(A) \to T(B) \\
\mathcal{F}_i^p & ::= & \mathcal{F}_i^p\ \&\ \mathcal{F}_i^p & T(A\ \&\ B) & = & T(A)\ \&\ T(B) \\
\mathcal{F}_i^p & ::= & \mathcal{F}_i^p \oplus \mathcal{F}_i^p & T(A \oplus B) & = & T(A) + T(B) \\
\mathcal{F}_i^p & ::= & \Box \mathcal{F}_i^p & T(\Box A) & = & \mathbf{L}T(A) \\
\mathcal{F}_i{}^o & ::= & \mathcal{F}_i{}^o - \mathcal{F}_i{}^o & T(A - B) & = & T(A)
\end{array}
$$

Fig. 6. Connectives and type map.

$$\frac{}{\overrightarrow{A} : x \Rightarrow A : x}\ id$$

$$\frac{\Gamma \Rightarrow A : \phi \quad \Delta\langle \overrightarrow{C} : z\rangle \Rightarrow D : \omega}{\Delta\langle \Gamma, \overrightarrow{A\backslash C} : y\rangle \Rightarrow D : \omega[(y\ \phi)/z]}\ \backslash L \qquad \frac{\overrightarrow{A} : x, \Gamma \Rightarrow C : \chi}{\Gamma \Rightarrow A\backslash C : \lambda x\chi}\ \backslash R$$

$$\frac{\Gamma \Rightarrow B : \psi \quad \Delta\langle \overrightarrow{C} : z\rangle \Rightarrow D : \omega}{\Delta\langle \overrightarrow{C/B} : x, \Gamma\rangle \Rightarrow D : \omega[(x\ \psi)/z]}\ /L \qquad \frac{\Gamma, \overrightarrow{B} : y \Rightarrow C : \chi}{\Gamma \Rightarrow C/B : \lambda y\chi}\ /R$$

$$\frac{\Delta\langle \overrightarrow{A} : x, \overrightarrow{B} : y\rangle \Rightarrow D : \omega}{\Delta\langle \overrightarrow{A \bullet B} : z\rangle \Rightarrow D : \omega[\pi_1 z/x, \pi_2 z/y]}\ \bullet L \qquad \frac{\Gamma_1 \Rightarrow A : \phi \quad \Gamma_2 \Rightarrow B : \psi}{\Gamma_1, \Gamma_2 \Rightarrow A \bullet B : (\phi, \psi)}\ \bullet R$$

$$\frac{\Gamma \Rightarrow A : \phi \quad \Delta\langle \overrightarrow{C} : z\rangle \Rightarrow D : \omega}{\Delta\langle \Gamma|_k \overrightarrow{A \downarrow_k C} : y\rangle \Rightarrow D : \omega[(y\ \phi)/z]}\ \downarrow_k L \qquad \frac{\overrightarrow{A} : x|_k \Gamma \Rightarrow C : \chi}{\Gamma \Rightarrow A \downarrow_k C : \lambda x\chi}\ \downarrow_k R$$

$$\frac{\Gamma \Rightarrow B : \psi \quad \Delta\langle \overrightarrow{C} : z\rangle \Rightarrow D : \omega}{\Delta\langle \overrightarrow{C \uparrow_k B} : x|_k \Gamma\rangle \Rightarrow D : \omega[(x\ \psi)/z]}\ \uparrow_k L \qquad \frac{\Gamma|_k \overrightarrow{B} : y \Rightarrow C : \chi}{\Gamma \Rightarrow C \uparrow_k B : \lambda y\chi}\ \uparrow_k R$$

$$\frac{\Gamma \Rightarrow A : \phi \quad \Delta\langle A : x; B : y\rangle \Rightarrow D : \omega}{\Delta\langle \Gamma; B|A : z\rangle \Rightarrow D : \omega[\phi/x, (z\ \phi)/y]}\ |L$$

$$\frac{\Gamma\langle B_0 : y_0; \ldots; B_n : y_n\rangle \Rightarrow D : \omega}{\Gamma\langle B_0|A : z_0; \ldots; B_n|A : z_n\rangle \Rightarrow D|A : \lambda x(\omega[(z_0\ x)/y_0, \ldots, (z_n\ x)/y_n])}\ |R$$

$$\frac{\Gamma\langle \overrightarrow{A} : x\rangle \Rightarrow B : \psi}{\Gamma\langle \overrightarrow{\Box A} : z\rangle \Rightarrow B : \psi[{}^\vee z/x]}\ \Box L \qquad \frac{\Box\Gamma \Rightarrow A : \phi}{\Box\Gamma \Rightarrow \Box A : {}^\wedge\phi}\ \Box R$$

$$\frac{\Gamma \Rightarrow A : \phi \quad \nvdash \Gamma \Rightarrow B : \_}{\Gamma \Rightarrow A - B : \phi}\ -R$$

Fig. 7. Semantically labelled sequent calculus for categorial logic, where $k \in \{\rangle, \langle\}$.

Proof. By induction on the length of derivations of **DCA**$\Box$ plus the difference operator. All the rules except for the difference rule assign trivially a Curry–Howard term. The (right) rule of '−' is non-standard for one of the premises has a not provable sequent:

$$\frac{\Delta \Rightarrow A : \Phi \quad \nvdash \Delta \Rightarrow B : \_}{\Delta \Rightarrow A - B : \Phi}\ -R$$

Here, by induction hypothesis the left premise sequent has a Curry–Howard term Φ. The other non-provable sequent does not matter because in the conclusion the succedent is assigned Φ. Therefore the '−' assigns a Curry–Howard term. This ends the proof. $\Box$

5. Anaphora

We assume basic types N for name or (referring) nominal, S for statement or (declarative) sentence, and CN for common (or count) noun. We assume the account of modality for intensionality of Morrill (see [21], [26, Chapter 5], [27, Chapter 8]) so that all lexical types are modalized outermost (i.e. the meanings of words are senses/intensions) and subordinate sentences are modalized (i.e. denote propositions). Thus for example the transitive verb *likes* will be of type $\Box((N\backslash S)/N)$ and the propositional attitude verb *believes* will be of type $\Box((N\backslash S)/\Box S)$. A verb phrase such as *likes Mary* will have type $N\backslash S$. An expression such as *believes John likes* will have type $(N\backslash S)/\Box N$ but not type $(N\backslash S)/N$ since the propositional attitude verb requires its dependent clause to be modal.

5.1. Possessive pronouns

A possessive pronoun *his/her/its* cannot take its antecedent from within its common noun complement:

$$*\text{His}_i \text{ friend of John}_i \text{ left} \tag{23}$$

However, it can take its antecedent intrasententially from anywhere outside its noun phrase, or intersententially, or extralinguistically:

a. John/everyone$_i$ saw his$_{i/j}$ neighbour
b. His$_{i/j}$ neighbour saw John$_i$ (24)

We assign[10]:

$$\textbf{his/her/its} : \Box((\Box N|N)/\Box CN) : poss \tag{25}$$

(Agreement features will be a straightforward final addition.)

5.2. Reflexive pronouns

Reflexive pronouns such as *himself/herself/itself* can take subject antecedents or object antecedents.

Subject-oriented reflexivization like

$$\text{John}_i \text{ buys himself}_i \text{ coffee} \tag{26}$$

is generated by assignment as follows, where here and throughout VP abbreviates $N\backslash S$, and as remarked earlier we allow ourselves to omit the subscript $\rangle$ for leftmost (indeed, here unique) discontinuity:

$$\textbf{himself/herself/itself} : \Box((VP \uparrow N) \downarrow VP) : {}^\wedge\lambda x\lambda y(x\ y\ y) \tag{27}$$

The hypothetical subtype is not modalized, ensuring that the antecedent is clause-local (cf. Principle A of [4]):

$$*\text{John}_i \text{ believes Mary likes himself}_i \tag{28}$$

Consider the following contrast:

a. John$_i$ likes the picture of himself$_i$
b. *John$_i$ likes the neighbour of himself$_i$ (29)

This is captured if we assume that in (29a) the prepositional phrase is a subcategorized complement but that in (29b) it is an adjunct with a modalized object as follows:

neighbour: $\Box CN$
of: $\Box(PP/N)$
of: $\Box((CN\backslash CN)/\Box N)$
picture: $\Box(CN/PP)$ (30)

For object-oriented reflexivization such as

$$\text{John talked to Mary}_i \text{ about herself}_i \tag{31}$$

[10] This fails to block a weak crossover violation such as *His$_i$ *neighbour saw everyone*$_i$.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

we assume assignment:

$$\textbf{himself/herself/itself}: \Box(((VP \uparrow N) \uparrow_{\langle} N) \downarrow_{\langle} (VP \uparrow N)): {}^{\wedge}\lambda x \lambda y(x\ y\ y) \tag{32}$$

This embodies a precedence condition on object-oriented reflexivization:

$$*\text{Mary revealed himself}_i \text{ to John}_i \tag{33}$$

The fact that the antecedent hypothetical subtype is not modalized prevents a clause non-local antecedent:

$$*\text{Mary notified the fact that John}_i \text{ won to himself}_i \tag{34}$$

With the assignments of (30) example (35a) is successfully blocked; however (35b) is overgenerated:

a. $*$Mary introduced the neighbour of John$_i$ to himself$_i$
b. $*$Mary showed the picture of John$_i$ to himself$_i$ (35)

5.3. Personal pronouns

We distinguish "external anaphora" in which the antecedent is intrasentential but outside the clause of the pronoun, or intersentential or extralinguistic, and "internal anaphora" in which the antecedent is within the clause of the pronoun or within a clause subordinate to that clause.

We assign to the nominative personal pronouns *he/she* as follows:

$$\textbf{he/she}: \Box((\Box S|N)/\Box VP): {}^{\wedge}\lambda x \lambda y^{\wedge}({}^{\vee}x\ y) \tag{36}$$

This captures that nominative pronouns only appear in subject positions, and that they permit no internal anaphora (cf. Principle C of [4]):

a. $*$He$_i$ likes John$_i$
b. $*$He$_i$ believes John$_i$ flies
c. $*$He$_i$ believes Mary likes John$_i$ (37)

To the both nominative and accusative personal pronoun *it* we assign for external anaphora thus:

$$\textbf{it}: \Box(\Box(S \uparrow N) \downarrow (\Box S|N)): {}^{\wedge}\lambda x \lambda y^{\wedge}({}^{\vee}x\ y) \tag{38}$$

This allows it to appear in both nominative and accusative positions.

To the accusative pronouns *him/her* we assign for external anaphora:

$$\textbf{him/her}: \Box(\Box((S \uparrow N) - (J \bullet VP)) \downarrow (\Box S|N)): {}^{\wedge}\lambda x \lambda y^{\wedge}({}^{\vee}x\ y) \tag{39}$$

This represents that the case in English is configurational and that the default case is accusative: the use of the difference operator (i.e. negation as failure) allows the accusative pronouns to appear anywhere except in subject position. For example, $*$*John$_i$ thinks him$_i$ runs* blocks because $1 + \textbf{runs}$, although it is of type $(\Box)(S \uparrow N)$, is also of type $J \bullet VP$.

Finally, for internal anaphora we assign thus to the accusative personal pronouns *him/her/it*[11]:

$$\textbf{him/her/it}: \Box((((S \uparrow N) \uparrow \Box N) - (J \bullet (VP \uparrow N))) \downarrow_{\langle} (S \uparrow \Box N)): {}^{\wedge}\lambda x \lambda y(x\ y\ {}^{\vee}y) \tag{40}$$

A similar device as before limits the accusative pronouns to only non-subject positions. That the antecedent hypothetical subtype is modalized allows a non-clause-local internal antecedent (by contrast with the reflexivization (34)):

$$\text{The fact that Mary employed John}_i \text{ surprised him}_i \tag{41}$$

The type embodies a precedence constraint on internal anaphora:

$$*\text{Mary revealed him}_i \text{ to John}_i \tag{42}$$

And the negation ensures that the pronoun cannot take as antecedent the subject of its own clause (cf. Principle B of [4]):

$$*\text{John}_i \text{ likes him}_i \tag{43}$$

As noted by a reviewer the use of negation in (39) could be avoided by treating case as a syntactic feature, while its use in (40) is essential to capture a Principle B effect.

[11] As noted by a reviewer this allows what Büring [1] calls Binding out of DP: *Everyone$_i$'s mother loves him$_i$*. Dowty [6] proposes an accusative pronoun type assignment which in our formalism is $((N\backslash S) \uparrow N) \downarrow (N\backslash S)$: $\lambda x \lambda y(x\ y\ y)$ for subject antecedents. That approach requires additional types such as $(((N\backslash S)/N) \uparrow N) \downarrow ((N\backslash S)/N)$ for other kinds of antecedents, as observed by Dowty, and does not capture Binding out of DP or Principle B effects.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

about : $\Box(PPabout/NA) : \hat{}\lambda BB$
believes : $\Box((Nt(s(A))\backslash Sf)/\Box Sf) : believe$
buys : $\Box(((Nt(s(A))\backslash Sf)/NB)/NC) : buy$
coffee : $\Box Nt(s(n)) : coffee$
everyone : $\Box((SA{\uparrow}Nt(s(B))){\downarrow}SA) : \hat{}\lambda C\forall D[(\check{}person\ D) \to (C\ D)]$
he : $\Box((\Box SA|Nt(s(m)))/\Box(Nt(s(m))\backslash SA)) : \hat{}\lambda B\lambda C\hat{}(\check{}B\ C)$
her : $\Box((((SA{\uparrow}Nt(s(f))){\uparrow}\Box Nt(s(f))) - (J\bullet((Nt(s(f))\backslash SA){\uparrow}Nt(s(f))))){\downarrow}_{(}(SA{\uparrow}\Box Nt(s(f)))) : \hat{}\lambda B\lambda C((B\ C)\ \check{}C)$
herself : $\Box((((NA\backslash SB){\uparrow}Nt(s(f))){\uparrow}Nt(s(f))){\downarrow}_{(}((NA\backslash SB){\uparrow}Nt(s(f)))) : \hat{}\lambda C\lambda D((C\ D)\ D)$
him : $\Box(\Box((SA{\uparrow}Nt(s(m))) - (J\bullet(Nt(s(m))\backslash SA))){\downarrow}(\Box SA|Nt(s(m)))) : \hat{}\lambda B\lambda C\hat{}(\check{}B\ C)$
himself : $\Box(((Nt(s(m))\backslash SA){\uparrow}Nt(s(m))){\downarrow}(Nt(s(m))\backslash SA)) : \hat{}\lambda B\lambda C((B\ C)\ C)$
his : $\Box((\Box Nt(A)|Nt(s(m)))/\Box CNA) : \hat{}\lambda B\lambda C\hat{}((\check{}of\ C)\ \check{}B)$
informs : $\Box(((Nt(s(A))\backslash Sf)/PPabout)/NB) : inform$
john : $\Box Nt(s(m)) : \hat{}j$
likes : $\Box((Nt(s(A))\backslash Sf)/NB) : like$
mary : $\Box Nt(s(f)) : \hat{}m$
neighbour : $\Box CNA : neighbour$
of : $\Box((CNA\backslash CNA)/\Box NB) : \hat{}\lambda C(\check{}of\ \check{}C)$
swims : $\Box(Nt(s(A))\backslash Sf) : swim$
the : $\Box(Nt(A)/CNA) : \iota$

Fig. 8. Lexicon for anaphora.

$$
\begin{prooftree}
\AxiomC{}
\UnaryInfC{$CNA \Rightarrow CNA$}
\RightLabel{$\Box L$}
\UnaryInfC{$\boxed{\Box CNA} \Rightarrow CNA$}
\RightLabel{$\Box R$}
\UnaryInfC{$\Box CNA \Rightarrow \Box CNA$}
\AxiomC{}
\UnaryInfC{$Nt(s(m)) \Rightarrow Nt(s(m))$}
\RightLabel{$\Box L$}
\UnaryInfC{$\boxed{\Box Nt(s(m))} \Rightarrow Nt(s(m))$}
\AxiomC{}
\UnaryInfC{$Nt(A) \Rightarrow Nt(A)$}
\RightLabel{$\Box L$}
\UnaryInfC{$\boxed{\Box Nt(A)} \Rightarrow Nt(A)$}
\AxiomC{}
\UnaryInfC{$Nt(s(m)) \Rightarrow Nt(s(m))$}
\AxiomC{}
\UnaryInfC{$Sf \Rightarrow Sf$}
\RightLabel{$\backslash L$}
\BinaryInfC{$Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf$}
\RightLabel{$/L$}
\BinaryInfC{$Nt(s(m)), \boxed{(Nt(s(m))\backslash Sf)/Nt(A)}, \Box Nt(A) \Rightarrow Sf$}
\RightLabel{$\Box L$}
\UnaryInfC{$Nt(s(m)), \boxed{\Box((Nt(s(m))\backslash Sf)/Nt(A))}, \Box Nt(A) \Rightarrow Sf$}
\RightLabel{$|L$}
\BinaryInfC{$\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(A)), \boxed{\Box Nt(A)|Nt(s(m))} \Rightarrow Sf$}
\RightLabel{$/L$}
\BinaryInfC{$\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(A)), \boxed{(\Box Nt(A)|Nt(s(m)))/\Box CNA}, \Box CNA \Rightarrow Sf$}
\RightLabel{$\Box L$}
\UnaryInfC{$\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(A)), \boxed{\Box((\Box Nt(A)|Nt(s(m)))/\Box CNA)}, \Box CNA \Rightarrow Sf$}
\end{prooftree}
$$

Fig. 9. Derivation for *John likes his neighbour.*

6. Exemplification

To exemplify the displacement logic and account of anaphora we assume the lexicon given in Fig. 8. Atomic types are structured with feature terms; free variables are interpreted as universally quantified at the outermost level and thus undergo unification.[12]

For the derivation of the possessive pronominalization **john** + **likes** + **his** + **neighbour** : S lexical lookup yields the semantically annotated sequent:

$$\Box Nt(s(m)) : \hat{}j, \Box((Nt(s(A))\backslash Sf)/NB) : like, \Box((\Box Nt(C)|Nt(s(m)))/\Box CNC) : \hat{}\lambda D\lambda E\hat{}((\check{}of\ E)\ \check{}D),$$
$$\Box CNF : neighbour \Rightarrow Sf \tag{44}$$

This has the proof given in Fig. 9, which delivers semantics:

$$((\check{}like\ ((\check{}of\ j)\ \check{}neighbour))\ j) \tag{45}$$

For the quantificational counterpart **everyone** + **likes** + **his** + **neighbour** : S there is the semantically annotated sequent:

$$\Box((SA{\uparrow}Nt(s(B))){\downarrow}SA) : \hat{}\lambda C\forall D[(\check{}person\ D) \to (C\ D)], \Box((Nt(s(E))\backslash Sf)/NF) : like,$$
$$\Box((\Box Nt(G)|Nt(s(m)))/\Box CNG) : \hat{}\lambda H\lambda I\hat{}((\check{}of\ I)\ \check{}H), \Box CNJ : neighbour \Rightarrow Sf \tag{46}$$

which has the derivational proof of Fig. 10.This delivers semantics:

$$\forall B[(\check{}person\ B) \to ((\check{}like\ ((\check{}of\ B)\ \check{}neighbour))\ B)] \tag{47}$$

[12] The sources for this section are computer generated by a Prolog parser/theorem-prover CatLog based on the principles described in [23].

$$\dfrac{\dfrac{\dfrac{}{CNA \Rightarrow CNA}}{\boxed{\Box CNA} \Rightarrow CNA}{\scriptstyle \Box L}}{\Box CNA \Rightarrow \Box CNA}{\scriptstyle \Box R} \quad \dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{Nt(A) \Rightarrow Nt(A)}}{\boxed{\Box Nt(A)} \Rightarrow Nt(A)}{\scriptstyle \Box L} \quad \dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{}{Sf \Rightarrow Sf}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}{\scriptstyle \backslash L}}{Nt(s(m)), \boxed{(Nt(s(m))\backslash Sf)/Nt(A)}, \Box Nt(A) \Rightarrow Sf}{\scriptstyle /L}}{Nt(s(m)), \boxed{\Box((Nt(s(m))\backslash Sf)/Nt(A))}, \Box Nt(A) \Rightarrow Sf}{\scriptstyle \Box L}}{Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(A)), \boxed{\Box Nt(A)|Nt(s(m))} \Rightarrow Sf}{\scriptstyle |L}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\cdots}{Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(A)), \boxed{(\Box Nt(A)|Nt(s(m)))/\Box CNA}, \Box CNA \Rightarrow Sf}{\scriptstyle /L}}{Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(A)), \boxed{\Box((\Box Nt(A)|Nt(s(m)))/\Box CNA)}, \Box CNA \Rightarrow Sf}{\scriptstyle \Box L}}{1, \Box((Nt(s(m))\backslash Sf)/Nt(A)), \Box((\Box Nt(A)|Nt(s(m)))/\Box CNA), \Box CNA \Rightarrow Sf\uparrow Nt(s(m))}{\scriptstyle \uparrow R} \quad \dfrac{}{Sf \Rightarrow Sf}}{\dfrac{\boxed{(Sf\uparrow Nt(s(m)))\downarrow Sf}, \Box((Nt(s(m))\backslash Sf)/Nt(A)), \Box((\Box Nt(A)|Nt(s(m)))/\Box CNA), \Box CNA \Rightarrow Sf}{\boxed{\Box((Sf\uparrow Nt(s(m)))\downarrow Sf)}, \Box((Nt(s(m))\backslash Sf)/Nt(A)), \Box((\Box Nt(A)|Nt(s(m)))/\Box CNA), \Box CNA \Rightarrow Sf}{\scriptstyle \Box L}}{\scriptstyle \downarrow L}$$

Fig. 10. Derivation for *Everyone likes his neighbour.*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{}{Sf \Rightarrow Sf}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}{\scriptstyle \backslash L}}{Nt(s(m)), \boxed{(Nt(s(m))\backslash Sf)/Nt(s(m))}, Nt(s(m)) \Rightarrow Sf}{\scriptstyle /L}}{Nt(s(m)), \boxed{\Box((Nt(s(m))\backslash Sf)/Nt(s(m)))}, Nt(s(m)) \Rightarrow Sf}{\scriptstyle \Box L}}{\Box((Nt(s(m))\backslash Sf)/Nt(s(m))), Nt(s(m)) \Rightarrow Nt(s(m))\backslash Sf}{\scriptstyle \backslash R}}{\Box((Nt(s(m))\backslash Sf)/Nt(s(m))), 1 \Rightarrow (Nt(s(m))\backslash Sf)\uparrow Nt(s(m))}{\scriptstyle \uparrow R} \quad \dfrac{\dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))}}{\boxed{\Box Nt(s(m))} \Rightarrow Nt(s(m))}{\scriptstyle \Box L} \quad \dfrac{}{Sf \Rightarrow Sf}}{\Box Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}{\scriptstyle \backslash L}}{\dfrac{\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(s(m))), \boxed{((Nt(s(m))\backslash Sf)\uparrow Nt(s(m)))\downarrow(Nt(s(m))\backslash Sf)} \Rightarrow Sf}{\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/Nt(s(m))), \boxed{\Box(((Nt(s(m))\backslash Sf)\uparrow Nt(s(m)))\downarrow(Nt(s(m))\backslash Sf))} \Rightarrow Sf}{\scriptstyle \Box L}}{\scriptstyle \downarrow L}$$

Fig. 11. Derivation of *John likes himself.*

For the derivation of the subject-oriented reflexivization **john** + **likes** + **himself** : S lexical lookup yields the semantically annotated sequent:

$$\Box Nt(s(m)) : \hat{}j, \Box((Nt(s(A))\backslash Sf)/NB) : like,$$
$$\Box(((Nt(s(m))\backslash SC)\uparrow Nt(s(m)))\downarrow(Nt(s(m))\backslash SC)) : \hat{}\lambda D\lambda E((D\ E)\ E) \Rightarrow Sf \tag{48}$$

This has the proof given in Fig. 11, which delivers semantics:

$$((\check{}like\ j)\ j) \tag{49}$$

For the quantificational counterpart **everyone** + **likes** + **himself** : S there is the semantically annotated sequent:

$$\Box((SA\uparrow Nt(s(B)))\downarrow SA) : \hat{}\lambda C\forall D[(\check{}person\ D) \rightarrow (C\ D)], \Box((Nt(s(E))\backslash Sf)/NF) : like,$$
$$\Box(((Nt(s(m))\backslash SG)\uparrow Nt(s(m)))\downarrow(Nt(s(m))\backslash SG)) : \hat{}\lambda H\lambda I((H\ I)\ I) \Rightarrow Sf \tag{50}$$

This has the proof of Fig. 12 which delivers semantics:

$$\forall B[(\check{}person\ B) \rightarrow ((\check{}like\ B)\ B)] \tag{51}$$

For the derivation of the verb phrase medial subject-oriented reflexivization **john** + **buys** + **himself** + **coffee** : S lexical lookup yields the semantically annotated sequent:

$$\Box Nt(s(m)) : \hat{}j, \Box(((Nt(s(A))\backslash Sf)/NB)/NC) : buy,$$
$$\Box(((Nt(s(m))\backslash SD)\uparrow Nt(s(m)))\downarrow(Nt(s(m))\backslash SD)) : \hat{}\lambda E\lambda F((E\ F)\ F), \Box Nt(s(n)) : coffee \Rightarrow Sf \tag{52}$$

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

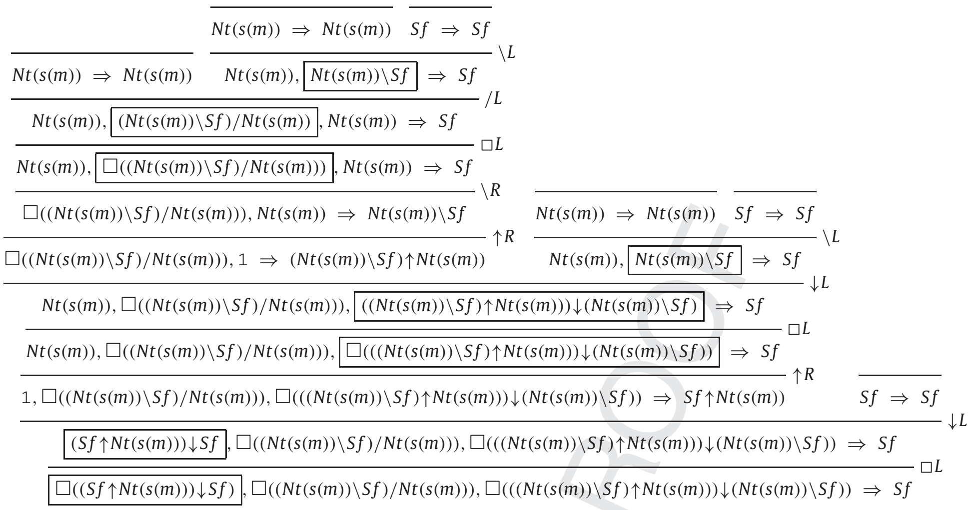

Fig. 12. Derivation of *Everyone likes himself*.

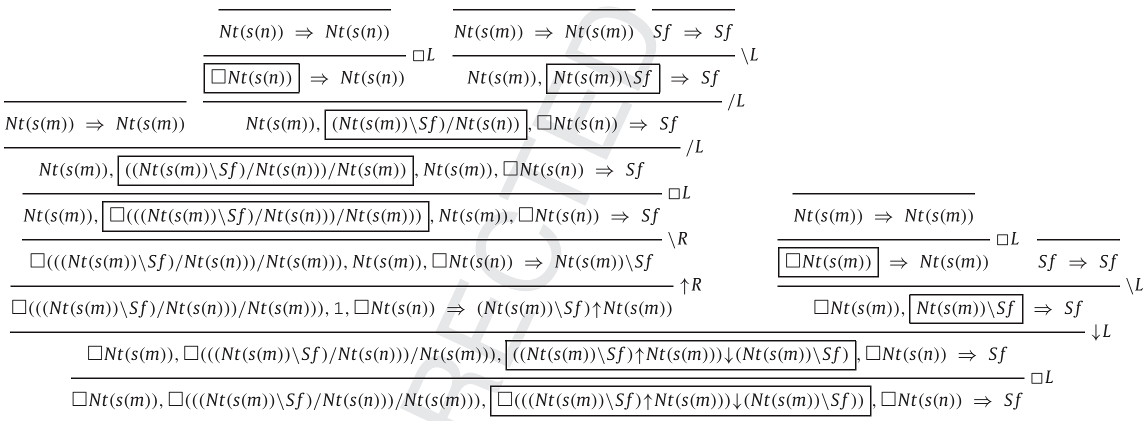

Fig. 13. Derivation for *John buys himself coffee*.

This has the derivation given in Fig. 13, which delivers semantics:

$$((({}^{\vee}buy\ j)\ {}^{\vee}coffee)\ j) \tag{53}$$

Principle A violations such as **john + believes + mary + likes + himself** : S and **mary + believes + john + likes + herself** : S have no derivation because the propositional attitude verb projects a modalized domain.

For the derivation of the object-oriented reflexivization **john + informs + mary + about + herself** : S lexical lookup yields the semantically annotated sequent:

$$\Box Nt(s(m)) : {}^{\wedge}j, \Box(((Nt(s(A))\backslash Sf)/PP\,about)/NB) : inform, \Box Nt(s(f)) : {}^{\wedge}m,$$
$$\Box(PP\,about/NC) : {}^{\wedge}\lambda D D, \Box((((NE\backslash SF)\uparrow Nt(s(f)))\uparrow Nt(s(f)))\downarrow_{\langle}((NE\backslash SF)\uparrow Nt(s(f)))) :$$
$${}^{\wedge}\lambda G\lambda H((G\ H)\ H) \Rightarrow Sf \tag{54}$$

This has the proof given in Fig. 14, which delivers semantics:

$$((({}^{\vee}inform\ m)\ m)\ j) \tag{55}$$

For the derivation of the external nominative pronominalization **john + believes + he + swims** : S lexical lookup yields:

$$\Box Nt(s(m)) : {}^{\wedge}j, \Box((Nt(s(A))\backslash Sf)/\Box Sf) : believe,$$
$$\Box((\Box SB|Nt(s(m)))/\Box(Nt(s(m))\backslash SB)) : {}^{\wedge}\lambda C\lambda D{}^{\wedge}({}^{\vee}C\ D), \Box(Nt(s(E))\backslash Sf) : swim \Rightarrow Sf \tag{56}$$

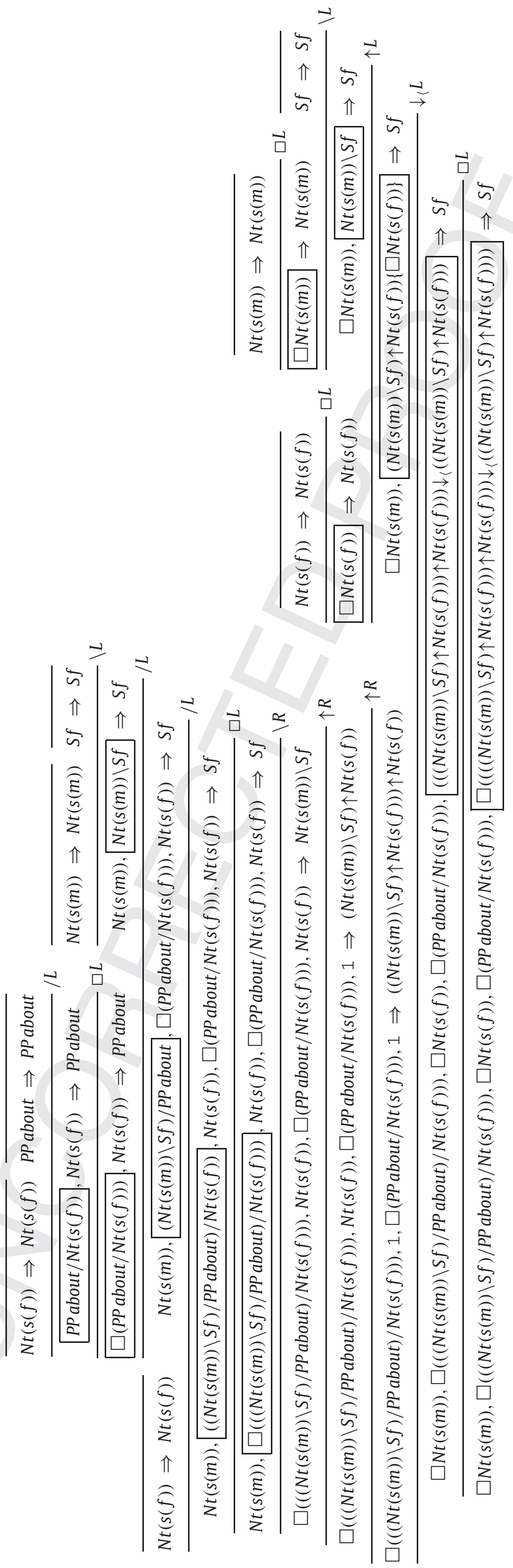

Fig. 14. Derivation of *John informs Mary about herself.*

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Sf \Rightarrow Sf}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}\backslash L}{Nt(s(m)), \boxed{\Box(Nt(s(m))\backslash Sf)} \Rightarrow Sf}\Box L}{\Box(Nt(s(m))\backslash Sf) \Rightarrow Nt(s(m))\backslash Sf}\backslash R}{\Box(Nt(s(m))\backslash Sf) \Rightarrow \Box(Nt(s(m))\backslash Sf)}\Box R
\quad
\dfrac{\dfrac{Nt(s(m)) \Rightarrow Nt(s(m))}{\boxed{\Box Nt(s(m))} \Rightarrow Nt(s(m))}\Box L
\quad
\dfrac{\dfrac{\dfrac{\dfrac{Sf \Rightarrow Sf}{\boxed{\Box Sf} \Rightarrow Sf}\Box L}{\Box Sf \Rightarrow \Box Sf}\Box R
\quad
\dfrac{Nt(s(m)) \Rightarrow Nt(s(m)) \quad Sf \Rightarrow Sf}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}\backslash L}{Nt(s(m)), \boxed{(Nt(s(m))\backslash Sf)/\Box Sf}, \Box Sf \Rightarrow Sf}/L}{Nt(s(m)), \boxed{\Box((Nt(s(m))\backslash Sf)/\Box Sf)}, \Box Sf \Rightarrow Sf}\Box L}{\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/\Box Sf), \boxed{\Box Sf|Nt(s(m))} \Rightarrow Sf}|L}
{\dfrac{\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/\Box Sf), \boxed{(\Box Sf|Nt(s(m)))/\Box(Nt(s(m))\backslash Sf)}, \Box(Nt(s(m))\backslash Sf) \Rightarrow Sf}{\Box Nt(s(m)), \Box((Nt(s(m))\backslash Sf)/\Box Sf), \boxed{\Box((\Box Sf|Nt(s(m)))/\Box(Nt(s(m))\backslash Sf))}, \Box(Nt(s(m))\backslash Sf) \Rightarrow Sf}\Box L}/L
$$

Fig. 15. Derivation of *John believes he swims.*

This has the proof given in Fig. 15, which delivers semantics:

$$((\check{}believe\ \hat{}(\check{}swim\ j))\ j) \tag{57}$$

For the external accusative pronominalization **john** + **believes** + **mary** + **likes** + **him** : S lexical lookup yields:

$$\begin{aligned}
&\Box Nt(s(m)) : \hat{}j, \Box((Nt(s(A))\backslash Sf)/\Box Sf) : believe, \Box Nt(s(f)) : \hat{}m,\\
&\Box((Nt(s(B))\backslash Sf)/NC) : like,\\
&\Box(\Box((SD{\uparrow}Nt(s(m))) - (J\bullet(Nt(s(m))\backslash SD))){\downarrow}(\Box SD|Nt(s(m)))) : \hat{}\lambda E\lambda F\hat{}(\check{}E\ F) \Rightarrow Sf
\end{aligned} \tag{58}$$

This has the proof given in Fig. 16. (For the difference operator it is impracticable to portray the search testifying to the non-existence of a proof for the negative subgoal, so only the positive subproof is represented in derivations.) This delivers semantics:

$$((\check{}believe\ \hat{}((\check{}like\ j)\ m))\ j) \tag{59}$$

For the derivation of the internal anaphora **the** + **neighbour** + **of** + **john** + **likes** + **him** : S lexical lookup yields the sequent:

$$\begin{aligned}
&\Box(Nt(A)/CNA) : \iota, \Box CNB : neighbour, \Box((CNC\backslash CNC)/\Box ND) : \hat{}\lambda E(\check{}of\ \check{}E),\\
&\Box Nt(s(f)) : \hat{}m, \Box((Nt(s(F))\backslash Sf)/NG) : like,\\
&\Box((((SH{\uparrow}Nt(s(f))){\uparrow}\Box Nt(s(f))) - (J\bullet((Nt(s(f))\backslash SH){\uparrow}Nt(s(f))))){\downarrow}_{\langle}(SH{\uparrow}\Box Nt(s(f)))) :\\
&\hat{}\lambda I\lambda J((I\ J)\ \check{}J) \Rightarrow Sf
\end{aligned} \tag{60}$$

This has the proof given in Fig. 17, which yields semantics:

$$((\check{}like\ m)\ (\check{}\iota\ ((\check{}of\ m)\ \check{}neighbour))) \tag{61}$$

7. Conclusion

Anaphora occurs widely in natural language and its analysis raises methodological challenges. Anaphora can be intersentential or deictic, and some of the generalizations governing it seem to require negative conditions which are not easy to express naturally in grammar which is a formal generative system.

In this paper we have given a categorial treatment of anaphora which distinguishes what we call external anaphora and internal anaphora. The minimal governing category (MGC) of a pronoun is the smallest clause or noun phrase within which the pronoun falls. In external anaphora the pronoun takes its antecedent from outside its MGC, with no precedence constraint, and we characterize this in terms of (a version of) the limited contraction of Jäger [11], which drives the semantic duplication of pronominalization syntactically. In internal anaphora the pronoun takes its antecedent from within its MGC, with a precedence constraint, and we characterize this in terms of the displacement calculus of Morrill, Valentín and Fadda [25], which drives the semantic duplication of pronominalization lexically.

Here, binding principle A (locality) effects on reflexive internal anaphora are approached by means of the modalization of Morrill [21]. Binding principle B (antilocality) on personal pronoun internal anaphora is modelled by employing the negation as failure of Morrill and Valentín [24]. Binding principle C effects follow from the fact that in our analysis only external anaphora can be cataphoric.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006

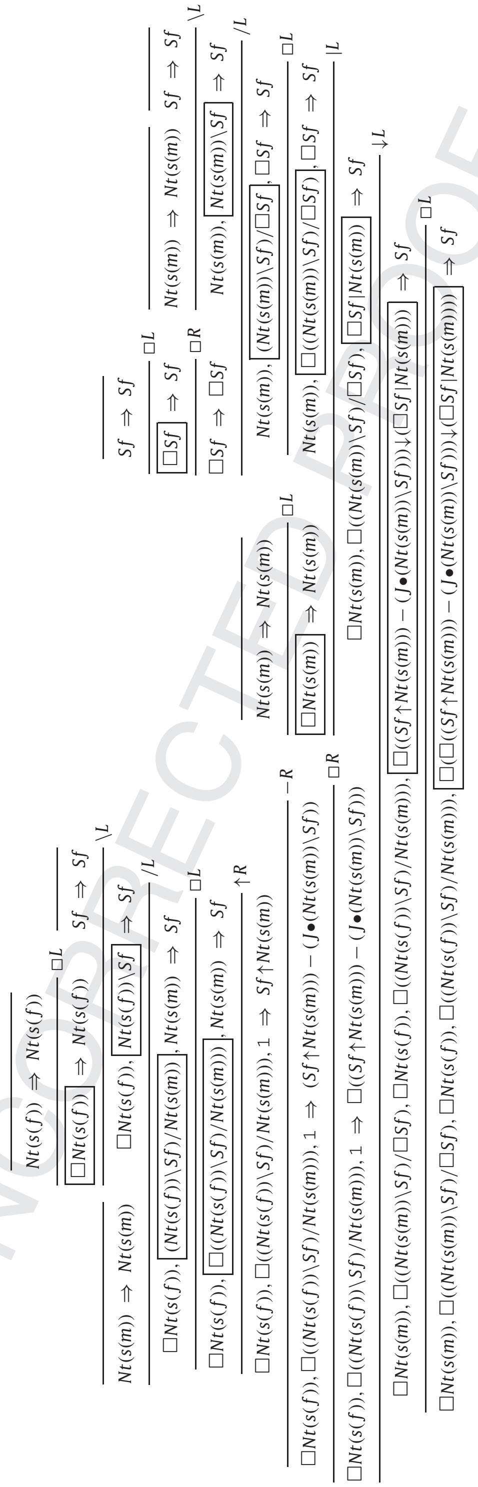

Fig. 16. Derivation of *John believes Mary likes him.*

Fig. 17. Derivation of *The neighbour of John likes him.*

Our characterization of the case distinction between the nominative pronouns *he*/*she* and the accusative pronouns *him*/*her* also uses the negation as failure and reflects the received wisdom that in English case is configurational and that the default case is accusative. The account dispenses with case features.

References

[1] D. Büring, Crossover situations, Nat. Lang. Semant. 12 (2004) 23–62.
[2] W. Buszkowski, Categorial grammars with negative information, in: A Notion in Focus, de Gruyter, 1995, pp. 107–126.
[3] B. Carpenter, Type-Logical Semantics, MIT Press, Cambridge, MA, 1997.
[4] N. Chomsky, Lectures on Government and Binding: The Pisa Lectures, Foris Publications, Dordrecht, 1981.
[5] K. Došen, Sequent systems and groupoid models, Part I, Studia Logica 47 (1988) 353–385.
[6] D. Dowty, Compositionality as an empirical problem, in: C. Barker, P. Jacobson (Eds.), Direct Compositionality, Oxford University Press, New York, 2007, pp. 23–101.
[7] J.Y. Girard, Linear logic, Theoret. Comput. Sci. 50 (1987) 1–102.
[8] P. de Groote, Towards abstract categorial grammars, in: Proceedings of the 39th Annual Meeting of the Association for Computational Linguistics (ACL), Toulouse, 2001, pp. 148–155.
[9] M. Hepple, The grammar and processing of order and dependency, PhD thesis, University of Edinburgh, 1990.
[10] P. Jacobson, Towards a variable-free semantics, Linguist. Philos. 22 (1999) 117–184.
[11] G. Jäger, Anaphora and Type Logical Grammar, Trends Log. Stud. Log. Libr., vol. 24, Springer, Dordrecht, 2005.
[12] M. Kanazawa, The Lambek calculus enriched with additional connectives, J. Log. Lang. Inf. 1 (1992) 141–171.
[13] J. Lambek, The mathematics of sentence structure, Amer. Math. Monthly 65 (1958) 154–170; reprinted in: Wojciech Buszkowski, Wojciech Marciszewski, Johan van Benthem (Eds.), Categorial Grammar, in: Linguist. Lit. Stud. East. Eur., vol. 25, John Benjamins, Amsterdam, 1988, pp. 153–172.
[14] J. Lambek, On the calculus of syntactic types, in: R. Jakobson (Ed.), Structure of Language and Its Mathematical Aspects, in: Proc. Sympos. Appl. Math., vol. XII, American Mathematical Society, Providence, RI, 1961, pp. 166–178.
[15] J. Lambek, Type grammars revisited, in: A. Lecomte, F. Lamarche, G. Perrier (Eds.), Logical Aspects of Computational Linguistics: Selected Papers from the Second International Conference, LACL '97, in: Lecture Notes in Artificial Intelligence, vol. 1582, Springer, NY, 1999, pp. 1–27.
[16] J. Lambek, From Word to Sentence. A Computational Algebraic Approach to Grammar, Polimetrica, 2008.
[17] V.W. Marek, Reflexive autoepistemic logic and logic programming, in: L.M. Pereira, A. Nerode (Eds.), Logic Programming and Non-Monotonic Reasoning, MIT Press, 1993.
[18] M. Moortgat, Categorial investigations: logical and linguistic aspects of the Lambek calculus, PhD thesis, Universiteit van Amsterdam, Foris, Dordrecht, 1988.
[19] M. Moortgat, Categorial type logics, in: J. van Benthem, A. ter Meulen (Eds.), Handbook of Logic and Language, Elsevier Science B.V./MIT Press, Amsterdam/Cambridge, MA, 1997, pp. 93–177.
[20] G. Morrill, Grammar and logical types, in: M. Stockhof, L. Torenvliet (Eds.), Proceedings of the Seventh Amsterdam Colloquium, 1990, pp. 429–450; also in: G. Barry, G. Morrill (Eds.), Studies in Categorial Grammar, in: Edinb. Work. Pap. Cogn. Sci., vol. 5, 1990, pp. 127–148; revised version published in: Grammar and Logic, Theoria LXII (3) (1996) 260–293.
[21] G. Morrill, Intensionality and boundedness, Linguist. Philos. 13 (1990) 699–726.
[22] G. Morrill, On bound anaphora in type logical grammar, in: G.J.M. Kruijff, R.T. Oehrle (Eds.), Resource-Sensitivity, Binding and Anaphora, in: Stud. Linguist. Philos., vol. 80, Kluwer Academic Press, Dordrecht, 2003, pp. 159–177.
[23] G. Morrill, Logic programming of the displacement calculus, in: S. Pogodalla, J.P. Prost (Eds.), Logical Aspects of Computational Linguistics: Proceedings of LACL'11, Montpellier, Springer-Verlag, Berlin, 2011, pp. 175–189.
[24] G. Morrill, O. Valentín, On anaphora and the binding principles in categorial grammar, in: A. Dawar, R. de Queiroz (Eds.), Proceedings of the 17th International Workshop on Logic, Language, Information and Computation, WoLLIC 2010, Brasília, Springer, Berlin, 2010, pp. 176–190.
[25] G. Morrill, O. Valentín, M. Fadda, The displacement calculus, J. Log. Lang. Inf. 20 (2011) 1–48, http://dx.doi.org/10.1007/s10849-010-9129-2.
[26] G.V. Morrill, Type Logical Grammar: Categorial Logic of Signs, Kluwer Academic Publishers, Dordrecht, 1994.
[27] G.V. Morrill, Categorial Grammar: Logical Syntax, Semantics, and Processing, Oxford University Press, 2011.
[28] R. Muskens, Lambda grammars and the syntax–semantics interface, in: R. van Rooy, M. Stokhof (Eds.), Proceedings of the Thirteenth Amsterdam Colloquium, Amsterdam, 2001, pp. 150–155.
Q2 [29] O. Valentín, Discontinuous Lambek calculus, PhD thesis, Universitat Autònoma de Barcelona, Barcelona, 2013, forthcoming.

Please cite this article in press as: G. Morrill, O. Valentín, Displacement logic for anaphora, J. Comput. System Sci. (2013), http://dx.doi.org/10.1016/j.jcss.2013.05.006